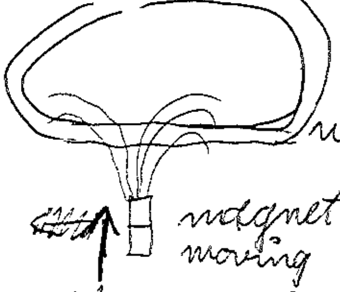
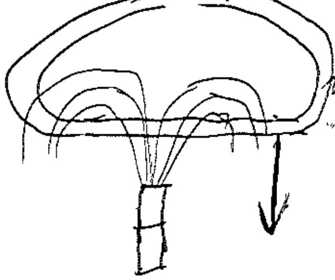
Special Relativity

[Goal: spacetime geometry! History...
new theory can't be deduced, motivation possible

Einstein (1909): „On the electrodynamics of moving bodies"
unnatural asymmetry in electrodynamics:



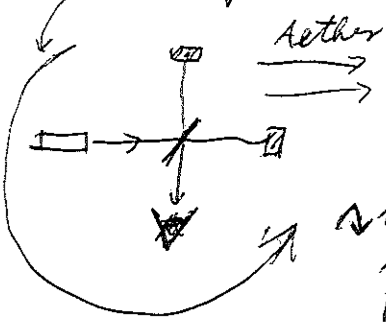
time-dep. B-field
↝ E-field in wire
↝ current



no E-field, but
Lorentz-force in wire
↝ same current as before!

Also: Michelson-Morley experiment

Interferometer
- moving through Aether
- rotating



[repeated half a year later]

↝ no change in interference pattern! ↝ $\boxed{\text{no Aether}}$!?

↳ **Principle of special relativity!**
in all inertial systems, where the mechanical laws hold, the electrodynamic equations also hold
↳ no Aether
↳ speed of light c in all inertial systems is the same

[extra assumption, could change Edyn]

[in GR: all laws of physics!]

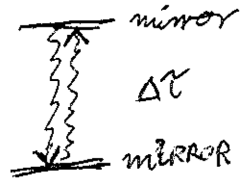
useful units: $c=1$

$\curvearrowright$ $1s \,\hat{=}\, 3\cdot 10^5 km \approx 80\%$ distance Earth-Moon

time dilation

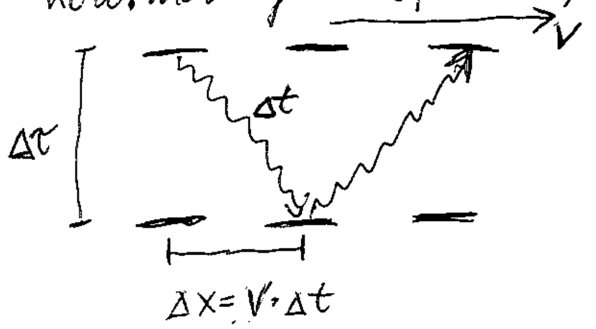
time is what a clock measures!

a clock based on light-travel:



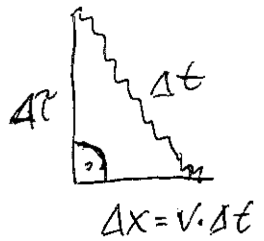
1 tick of the clock $= 2\Delta\tau$
$\hat{=}$ photon travel time
$\hat{=}$ 2· distance between mirrors ($c=1$)

now: moving clock, velocity V



$2\cdot\Delta t$: photon travel time

$\Delta t \neq \Delta\tau$ since photon travels a longer distance!

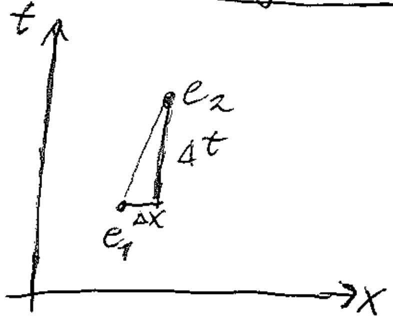


$$\Delta t^2 = \Delta\tau^2 + \underbrace{\Delta x^2}_{V^2\Delta t^2} \longrightarrow \boxed{\text{Important: } \Delta\tau^2 = \Delta t^2 - \Delta x^2}$$

$\hookrightarrow \frac{\Delta t}{\Delta\tau} = \gamma = \frac{1}{\sqrt{1-V^2}} \geq 1$ "gamma factor"

$\hookrightarrow \Delta t \geq \Delta\tau$ time dilation

spacetime diagrams



events: points in spacetime
e_1: breakfast on Earth
e_2: dinner on the moon

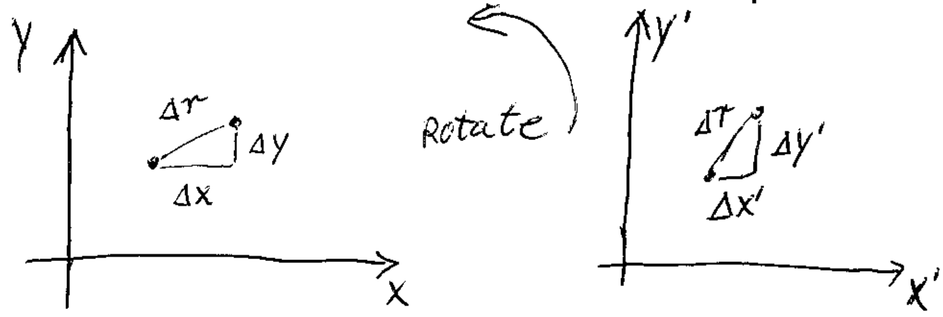
$\Delta\tau$: travel time in rest-system
= proper time (between e_1 and e_2)

$$\underline{\Delta\tau^2 = \Delta t^2 - \Delta x^2 = \text{invariant!}}$$

- $\Delta x, \Delta t$ are observer-dependent
- another observer measures $\Delta x', \Delta t'$

but: $\Delta\tau^2 \equiv \Delta t^2 - \Delta x^2 = \Delta t'^2 - \Delta x'^2$ = invariant!

analogous to distance in Euclidean space:



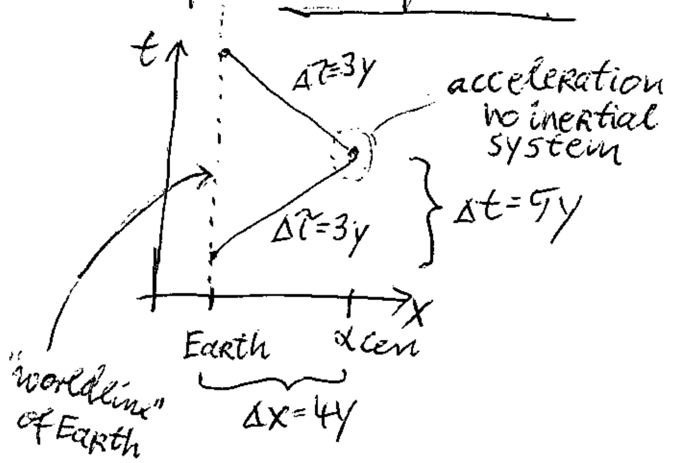
$\Delta\tau^2 = \Delta x^2 + \Delta y^2$
$= \Delta x'^2 + \Delta y'^2$
= invariant under rotation of coordinate-system

$\boxed{\text{$\Delta\tau$ is an invariant "distance" between events in spacetime!}}$

in general: $\Delta\tau^2 = \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$ "spacetime-Pythagoras"
<u>line element</u>

minus!

example: <u>twin paradox</u>



$\Delta\tau^2 = \Delta t^2 - \Delta x^2$
$= 25 - 16 = 9$

$\Rightarrow \Delta\tau = 3y \Rightarrow$ time in spaceship running slower!

in spaceship:

$\Delta\tau^2 = \Delta t'^2 - \underbrace{\Delta x'^2}_{0}$

$\Delta\tau = \Delta t'$ = proper time

paradox: seen from the spaceship, time on Earth is running slower.

Resolution: during acceleration, time on Earth must catch up.

$\boxed{\text{the geometric point of view gives clear answers $\rightsquigarrow$ no paradox!}}$

advice: draw spacetime diagram in <u>inertial frames</u>

Remark: <u>acceleration</u> reduces proper time

Lorentz-contraction

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How can the spaceship travel $\Delta x = 4y$ in $\Delta\tau = 3y$?

↳ distance Earth-α-Cen must be contracted as seen from the spaceship

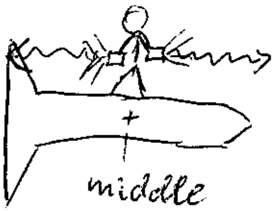
$\Delta\tau = \frac{\Delta t}{\gamma}$, distance seen from spaceship $= L'$, $\Delta x = L$

$L' = v \cdot \Delta\tau = \frac{v \cdot \Delta t}{\gamma} = \frac{L}{\gamma}$,

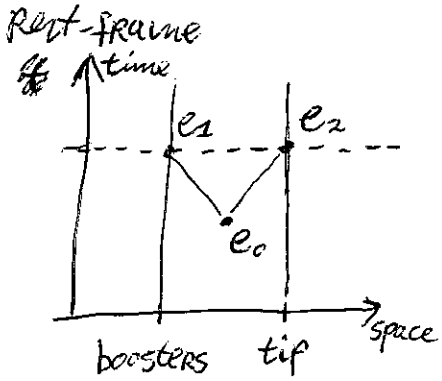
$\gamma \geq 1 \Rightarrow L' \leq L$ length contraction

Relativity of simultaneity

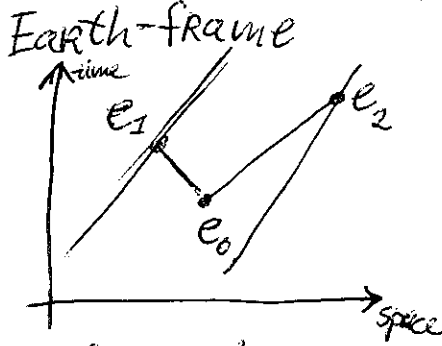
[Imagine L' is contracted to length of the ship. Ship contracted seen from Earth. How?



e_0: light bulbs switched on



e_1 & e_2 simultaneous



e_1 & e_2 not simultaneous!

proper length Δs: length measured in rest-frame of the object

abstract: proper length between two events
= length measured in system where events are simultaneous

we argued: $\Delta t^2 - \Delta x^2 =$ invariant

in rest-frame of an object: $\Delta t = 0$ (simultaneous)
and $\Delta x = \Delta s$

$\Rightarrow \Delta s^2 = \Delta x^2 - \Delta t^2 =$ invariant

conclusions:

events: points in spacetime

invariant "distance" between events:

line element $\Delta t^2 - \Delta x^2 = \begin{cases} \Delta\tau^2 & \text{for } \Delta t > \Delta x \\ -\Delta s^2 & \text{for } \Delta t < \Delta x \end{cases}$

$\boxed{\Rightarrow \text{spacetime geometry!}}$