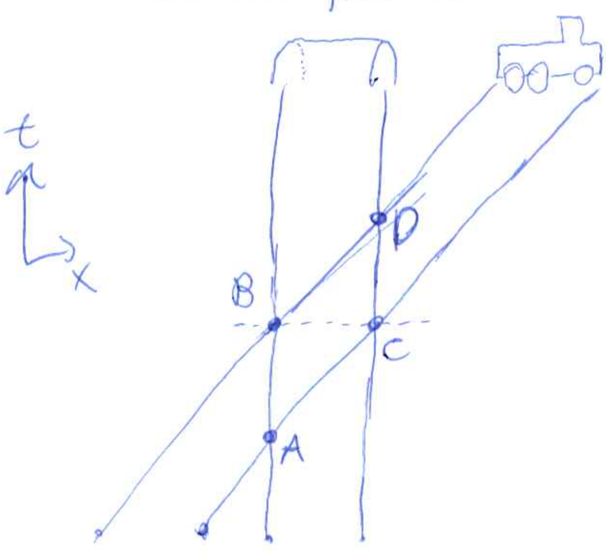
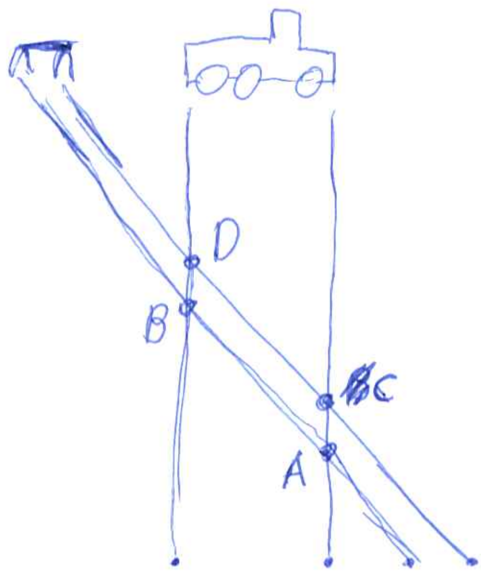
1.1.) tunnel frame

IMPRS 2018 ex P1





A: front of truck enters tunnel

B: back "

C: front of truck leaves tunnel

D: back "

The events B and C are simultaneous in the tunnel frame, i.e. all of the truck is simultaneous in the tunnel. But in the truck frame, first C and then B happens (front leaves tunnel before the back enters).

↳ no paradox in the spacetime geometry, events simultaneous in the tunnel frame do not have to be simultaneous in the truck frame.

1.2.) When B and C happen simultaneously in the tunnel frame, the intact (Lorentz contracted) truck is locked in the tunnel. Immediately afterwards, it crashes and becomes physically contracted to 10m or less due to the crash.

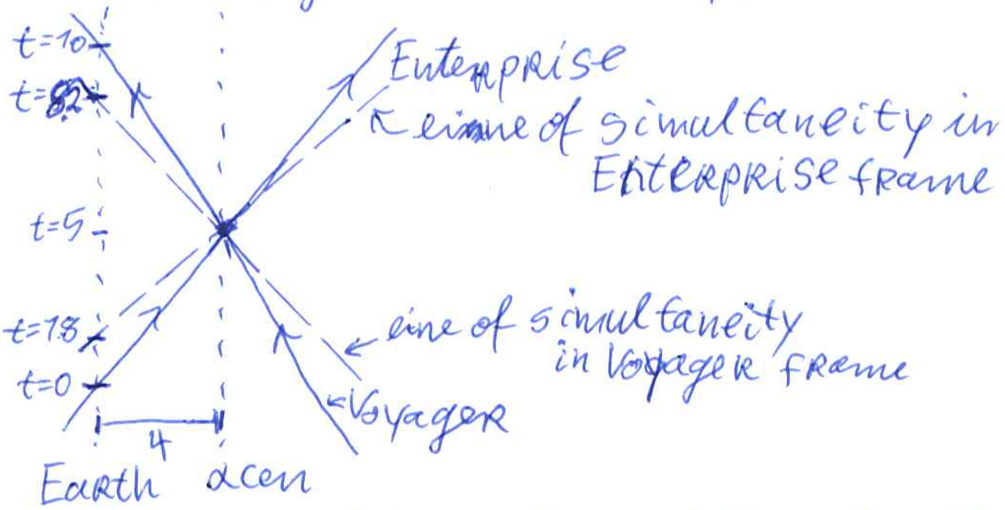
In the truck frame, the front crashes into the brick wall before the back is in the tunnel. However, this information can not be transmitted faster than light (e.g. through the interaction of the truck's atoms): It does not get there before all of the truck is in the tunnel.

IMPRS 2018 ex P2

2.1.) M: space ships meet at α Cen
velocity of Enterprise w.r.t Earth: $v = \frac{4}{5}$, $\gamma = \frac{5}{3}$
$\rightsquigarrow$ travel time of Enterprise in its rest frame: 3y
$\rightsquigarrow$ time on stop watch at M: $3/\gamma = 9/5 = 1.8$ y

2.2.) Similar: travel time from M to Earth in Voyager frame: 3y, $v = \frac{4}{5}$, $\gamma = \frac{5}{3}$
$\rightsquigarrow$ time on stop watch at M: $10 - 3/\gamma = 8.2$ y

2.3.) spacetime diagram in Earth system:



$\rightsquigarrow$ Enterprise and Voyager have different notions of simultaneity
$\rightsquigarrow$ they observe different times on the stop watch simultaneous to M (lines of simultaneity in space time diagram)

2.4.) As the astronaut jumps, the stop watch jumps forward from 1.8y to 8.2y. If he accelerates continuously, then the jump in time on the stop watch is smoothed (= many small jumps).

3.1.) Due to the principle of special relativity, there is no change in Einstein's mirror image that he could observe when he is running.

Similarly, due to the equivalence principle, there is no change in his mirror image when he falls through the black hole horizon.

IMPRS2018 ex P4

4.) $\vec{\partial}_\theta = \frac{\partial \vec{q}}{\partial \theta} = r \begin{pmatrix} \cos\theta \, \cos\phi \\ \cos\theta \, \sin\phi \\ -\sin\theta \end{pmatrix}$ $\vec{\partial}_\phi = \frac{\partial \vec{q}}{\partial \phi} = r \begin{pmatrix} -\sin\theta \, \sin\phi \\ \sin\theta \, \cos\phi \\ 0 \end{pmatrix}$

$\vec{\partial}_\theta \cdot \vec{\partial}_\theta = r^2 \left(\cos^2\theta \cdot \underbrace{(\cos^2\phi + \sin^2\phi)}_{1} + \sin^2\theta \right) = r^2$

$\vec{\partial}_\phi \cdot \vec{\partial}_\phi = r^2 \left(\sin^2\theta \cdot (\sin^2\phi + \cos^2\phi) \right) = r^2 \cdot \sin^2\theta$

$\vec{\partial}_\theta \cdot \vec{\partial}_\phi = r^2 \left(-\cos\theta \sin\theta \cos\phi \sin\phi + \cos\theta \sin\theta \sin\phi \cos\phi \right) = 0$

$\curvearrowright (g_{\mu\nu}) = r^2 \begin{array}{cc} \theta & \phi \end{array} \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix} \begin{array}{c} \theta \\ \phi \end{array}$ $(g^{\mu\nu}) = \frac{1}{r^2} \begin{pmatrix} 1 & 0 \\ 0 & \sin^{-2}\theta \end{pmatrix}$

$\curvearrowright$ only nonvanishing derivative: $\partial_\theta g_{\phi\phi} = 2r^2 \sin\theta \cos\theta$

$\curvearrowright \Gamma^\theta{}_{\phi\phi} = \frac{1}{2} g^{\theta\theta} (-\partial_\theta g_{\phi\phi}) = -\sin\theta \cos\theta$

$\Gamma^\phi{}_{\theta\phi} = \Gamma^\phi{}_{\phi\theta} = \frac{1}{2} g^{\phi\phi} \partial_\theta g_{\phi\phi} = \frac{\cos\theta}{\sin\theta}$ all other ZERO

5.) Due to symmetries: $R_{\theta\phi\theta\phi} = -R_{\theta\phi\phi\theta} = -R_{\phi\theta\theta\phi} = R_{\phi\theta\phi\theta}$ all other zero

$R_{\theta\phi\theta\phi} = g_{\theta\theta} \left(\partial_\theta \Gamma^\theta{}_{\phi\phi} - \underbrace{\partial_\phi \Gamma}_{0} + \Gamma^\theta{}_{\phi\phi} \underbrace{\Gamma^\theta{}_{\theta\theta}}_{0} - \Gamma^\phi{}_{\phi\theta} \Gamma^\theta{}_{\phi\phi} \right)$

$= r^2 \left(-\cos^2\theta + \sin^2\theta + \cos^2\theta \right) = \underline{\underline{r^2 \sin^2\theta}}$

$R_{\phi\phi} = g^{\theta\theta} R_{\theta\phi\theta\phi}$
$= \frac{1}{r^2} \cdot r^2 \sin^2\theta$

$R_{\theta\theta} = g^{\phi\phi} R_{\phi\theta\phi\theta} = 1$

$R_{\theta\phi} = \underbrace{g^{\phi\theta}}_{0} R_{\phi\theta\theta\phi} = 0$

$\curvearrowright \boxed{R_{\mu\nu} = \frac{1}{r^2} g_{\mu\nu}}$

$R = g^{\mu\nu} R_{\mu\nu} = \frac{1}{r^2} \underbrace{g^{\mu\nu} g_{\mu\nu}}_{\delta^\mu{}_\mu = 2} = \frac{2}{r^2}$