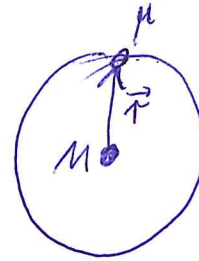
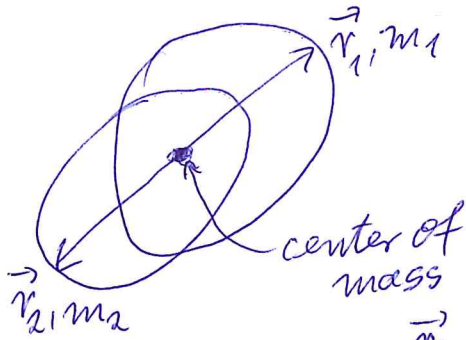


example: binary system in circular orbit
Newtonian estimate

JESS 2019 L2P1



$$\mu = \frac{m_1 m_2}{M}$$

$$M = m_1 + m_2$$

$$\vec{r}_1 = \frac{m_2}{M} \vec{r}, \quad \vec{r}_2 = -\frac{m_1}{M} \vec{r}, \quad \vec{r} = \vec{r}_1 - \vec{r}_2 = r \begin{pmatrix} \cos \phi \\ \sin \phi \\ 0 \end{pmatrix}$$

here: circular orbit, $\phi = \omega t$, $r = \text{const.}$

point-masses: $T_{00} \approx m_1 \delta(\vec{x} - \vec{r}_1) + m_2 \delta(\vec{x} - \vec{r}_2)$

$$\begin{aligned} \underline{\underline{M^{ij}}} &= m_1 \dot{r}_1^i \dot{r}_1^j + m_2 \dot{r}_2^i \dot{r}_2^j \\ &= \underline{\underline{\mu \dot{r}^i \dot{r}^j}} \end{aligned}$$

components:

$$M^{11} = \mu r^2 \cos^2 \omega t = \frac{1}{2} \mu r^2 (1 + \cos 2\omega t)$$

$$M^{22} = \frac{1}{2} \mu r^2 (1 - \cos 2\omega t)$$

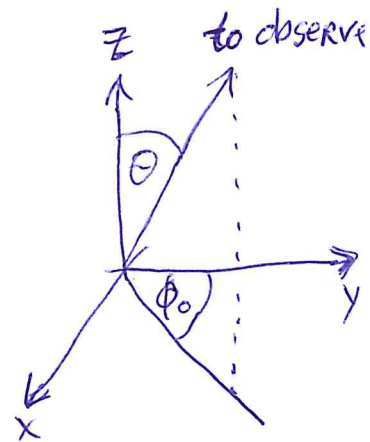
$$M^{12} = M^{21} = \frac{1}{2} \mu r^2 \sin 2\omega t \quad \text{other zero}$$

$\sim$ GW frequency = 2 x orbital frequency

... more calculations ...

$$\begin{aligned} h_+ &= - \frac{4\mu\omega^2 r^2}{R} \frac{1 + \cos^2 \theta}{2} \cos(2\omega t + 2\phi_0) \\ h_x &= - \frac{4\mu\omega^2 r^2}{R} \cos \theta \sin(2\omega t + 2\phi_0) \end{aligned}$$

amplitude
angular pattern
phase



chirp mass:

amplitude factor: $\mu \omega^2 r^2$ & 3rd Kepler $\omega^2 r^3 = M$

$$\hookrightarrow \mu \omega^2 r^2 = \mu \omega^2 \left(\frac{M}{\omega^2} \right)^{2/3} = \mu M^{2/3} \omega^{2/3} = M_c \omega^{2/3}$$

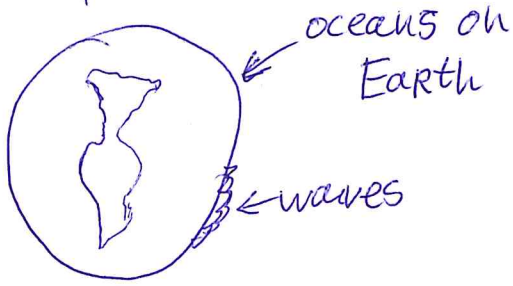
with $\boxed{M_c = \mu^{3/5} M^{2/5}}$ chirp mass!

$\hookrightarrow$ combination of masses that dominates GW properties

Gravitational waves on curved background

YESS 2019 L2P2

Example: waves on the ocean



which "part" of the water is the ocean and which is the wave
 → approximate notion
 1) small amplitude
 2) small wavelength / high frequency

GW: decompose

$$g_{\mu\nu} = \underbrace{g_{\mu\nu}^{LF}}_{\langle g_{\mu\nu} \rangle} + \underbrace{g_{\mu\nu}^{LF}}_{h_{\mu\nu}}$$

average

LF/HF: low/high freq. projection

decomposition of field eqs, schematically:

$$R = T \rightarrow R^{LF} + h + h^2 + \mathcal{O}(h^3) = T^{LF} + T^{HF} + \dots$$

$$\begin{aligned} &\xrightarrow{HF} \square h = T^{HF} + \mathcal{O}(h^2) \\ &\xrightarrow{LF} R^{LF} = \underbrace{T^{LF}}_{\langle T^{LF} \rangle} + \underbrace{(h^2)^{LF}}_{\langle h^2 \rangle} + \mathcal{O}(h^3) \\ &\text{projections} \quad \langle T^{LF} \rangle \quad \langle h^2 \rangle =: T_{\mu\nu}^{GW} \\ &\text{no } \mathcal{O}(h) \text{-term!} \end{aligned}$$

$$\boxed{T_{\mu\nu}^{GW} = \frac{1}{32\pi} \langle \partial_\mu h_{ij}^{TT} \partial_\nu h_{ij}^{TT} \rangle} \sim (\text{amplitude})^2$$

for $\langle T_{\mu\nu} \rangle = 0$, $\langle g_{\mu\nu} \rangle \approx \eta_{\mu\nu} + \mathcal{O}(h^2)$

GW carry energy and momentum

Quadrupole formula

Radiated power $\sim \frac{\text{energy loss}}{\text{time}}$

$\sim$ luminosity $\mathcal{L}$

energy-flow density: T_{GW}^{oi}

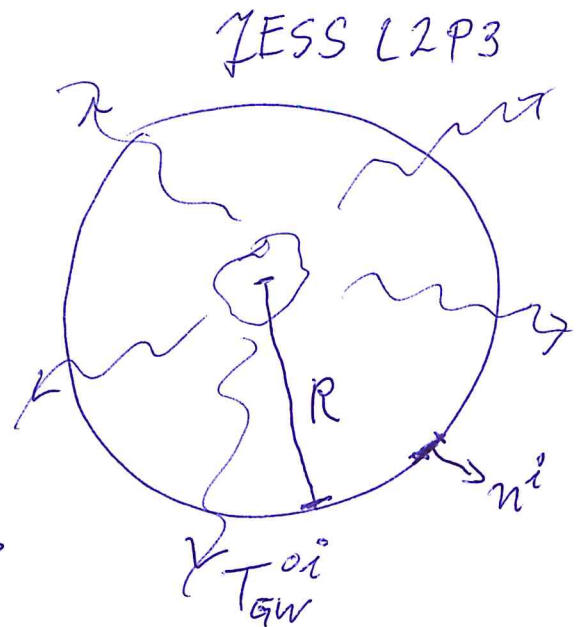
$$\mathcal{L} = \oint d\Omega \cdot R^2 n^i T_{GW}^{oi}$$

$$-T_{oi}^{GW} = -\frac{1}{32\pi} \langle \partial_0 h_{kl}^{\pi} \partial_i h_{kl}^{\pi} \rangle$$

$$h_{kl}^{\pi} \approx \frac{2}{R} \Lambda_{klif} \ddot{Q}^{ij}$$

$$\boxed{\mathcal{L} = \frac{1}{5} \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle}$$

quadrupole formula



for a binary, circular orbit:

$$\boxed{\mathcal{L} = \frac{32}{5} (M_c \omega)^{10/3}}$$

↑
chirp mass

(Monday)

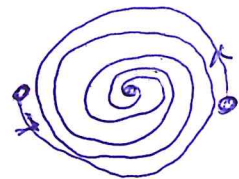
$$h \sim M_c^{5/3} \omega^{2/3}$$

$$\dot{h} \sim (M_c \omega)^{5/3}$$

$$\mathcal{L} \sim \dot{h}^2$$

energy loss $\rightarrow$ binary separation shrinks

$\rightarrow$ inspiral



energy balance:

$$\frac{dE}{dt} = -\mathcal{L} \quad (*)$$

$\left. \begin{array}{l} \uparrow r = \text{const} \\ \uparrow r \neq \text{const} \end{array} \right\}$ adiabatic approximation
orbit time $\ll$ radiation reaction time

$$E = \frac{1}{2} \mu v^2 - \frac{\mu M}{r}$$

$$= -\frac{1}{2} M_c^{5/3} \omega^{2/3}$$

$$v^2 = \omega^2 r^2 = \frac{M}{r}$$

↑
3rd Kepler

$$\text{in } (*): \dot{\omega} \cdot \omega^{-11/3} = \frac{32}{5} \cdot 3 \cdot M_c^{5/3} = \text{const}$$

ω increases $\rightarrow$ inspiral
ending in a merger!

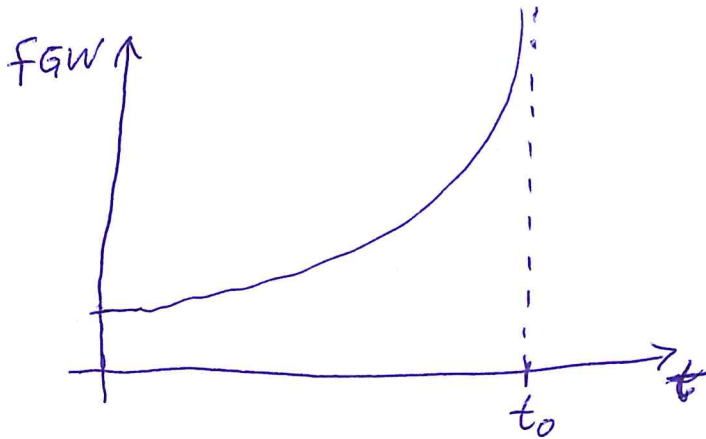
$\omega \rightarrow \infty$ at merger

integrate:

$$\omega^{-8/3} = \frac{256}{5} M_c^{5/3} \underbrace{(-t + t_0)}_{\tau: \text{time to merger}}$$

GW frequency $\omega_{\text{GW}} = 2\omega = 2\pi f_{\text{GW}}$

$$\rightarrow f_{\text{GW}} \simeq 134 \text{ Hz} \left(\frac{1.21 M_\odot}{M_c} \right)^{5/8} \left(\frac{15}{\tau} \right)^{3/8}$$

frequency evolution $\leadsto M_c$ GW150914: $M_c \simeq 30 M_\odot$

frequency goes up

& amplitude " "

 $\rightarrow$ like a chirp of birds