HU WS2016 V9S1

Gravitationswellen eines Binärsystems

allgemeiner Fall: Ü.blatt 7

↳ Exzentrizität nimmt stark ab durch Abstrahlung der GW

↳ Orbit wird näherungsweise Kreisförmig

daher ab jetzt: Kreisbahn $\boxed{\phi = \omega t}$ mit $\omega^2 r^3 = \underset{m_1+m_2}{\overset{\shortparallel}{M}}$ (3. Kepler'sches Gesetz)

brauchen Quadrupol $Q^{ij} = M^{ij} - \frac{1}{3}\delta^{ij} M^{kk}$

letzte VL:

$$M^{11} = \mu r^2 \cos^2\phi, \quad M^{22} = \mu r^2 \sin^2\phi, \quad M^{12} = M^{21} = \mu r^2 \cos\phi \sin\phi$$

für Punktmassen m_1 & m_2, $\mu = \frac{m_1 m_2}{m_1+m_2}$

$$\dot M^{11} = -2\mu r^2 \omega \sin\phi\cos\phi, \quad \ddot M^{11} = -2\mu r^2\omega^2 \underbrace{(\cos^2\phi - \sin^2\phi)}_{2\cos^2\phi - 1}, \quad \dddot M^{11} = \mu r^2 8\omega^3 \sin\phi\cos\phi$$

$$\dot M^{22} = +2\mu r^2 \omega \sin\phi\cos\phi, \quad \ddot M^{22} = + \quad '' \quad , \quad \dddot M^{22} = -\mu r^2 8\omega^3 \sin\phi\cos\phi$$

$$\dot M^{12} = \mu r^2 \omega \underbrace{(\cos^2\phi - \sin^2\phi)}_{2\cos^2\phi - 1}, \quad \ddot M^{12} = -\mu r^2 4\omega^2 \sin\phi\cos\phi, \quad \dddot M^{12} = -\mu r^2 4\omega^3 (2\cos^2\phi - 1)$$

Energieverlust:

$$\frac{dE}{dt} = -\frac{1}{5}\langle \dddot Q^{ij} \dddot Q^{ij}\rangle \leftarrow \text{Zeitmittelung}$$

$$= -\frac{1}{5}\langle \dddot Q^{ij}(\dddot M^{ij} - \tfrac{1}{3}\delta^{ij}\dddot M^{kk})\rangle \quad \to 0 \text{ da } \dddot Q^{ij}\delta^{ij} = 0$$

$$= -\frac{1}{5}\langle \dddot M^{ij}\dddot M^{ij} - \tfrac{1}{3}(\dddot M^{kk})^2\rangle$$

$$= -\frac{1}{5}\langle (\dddot M^{11})^2 + 2(\dddot M^{12})^2 + (\dddot M^{22})^2 - \tfrac{1}{3}(\dddot M^{kk})^2\rangle$$

$$= -\frac{1}{5}\underbrace{\int_{\text{Orbit}}\frac{dt}{T}}_{\int_0^{2\pi}\frac{d\phi}{2\pi}}(\mu r^2 \omega^3 4)^2 \underbrace{\left(8\sin^2\phi\cos^2\phi + 2(2\cos^2\phi - 1)^2\right)}_{8\cos^2\phi(1-\cos^2\phi) + 8\cos^4\phi - 8\cos^2\phi + 2 = 2}$$

mit $\dddot M^{kk} = \dddot M^{11} + \dddot M^{22} = 0$

$T = 2\pi/\omega$, $\phi = \omega t$

$$= -\frac{1}{5}(\mu r^2 4\omega^3)^2 \cdot 2 \underbrace{\int_0^{2\pi}\frac{d\phi}{2\pi}}_{1}$$

$$= -\frac{1}{10}\mu^2 r^4 (2\omega)^6 = -\mathcal{L}$$

Einsetzen: $r = (M\omega^{-2})^{1/3}$

$$\Rightarrow \frac{dE}{dt} = -\frac{32}{5}\mu^2 M^{4/3}\omega^{10/3} = -\frac{32}{5}(M_c\omega)^{10/3}$$

mit der "chirp"-Masse $M_c = \mu^{3/5} M^{2/5} = \left(\frac{m_1^3 m_2^3}{m_1+m_2}\right)^{1/5}$

Energieverlust

↳ r schrumpft } → adiabatisch

↳ ω steigt

↳ Intensität der GW $\mathcal{L}$ steigt } chirp: "zwitschern"

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berechne $\omega, \dot{r}$ ~~nach~~ in adiabatischer Näherung

$$E = E_{kin} + E_{pot} = \frac{1}{2}\mu v^2 - \frac{\mu M}{r}$$

mit $v^2 = \omega^2 r^2 \underset{\uparrow\,3.\,\text{Kepler}}{=} \frac{M}{r}$

$$E = -\frac{1}{2}\mu\omega^2 r^2 = -\frac{1}{2}\mu\frac{M}{r}\ , \quad \frac{dE}{dt} = -\cancel{\frac{32}{5}\mu M^{4/3}\omega^{10/3}} - \frac{32}{5}\mu^2 r^4 \omega^6 = -\frac{32}{5}\mu^2 M^3 r^{-5}$$

$$\leadsto -\frac{1}{2}\mu M \frac{dr^{-1}}{dt} = -\frac{32}{5}\mu^2 M^3 r^{-5}$$

$$r^3 \frac{dr}{dt} = -\frac{64}{5}\mu M^2$$

Integrieren:

$$\cancel{\tfrac{1}{4}}\, r^4 = +\frac{\cancel{64}\ 256}{5}\mu M^2 (t_0 - t)$$

↑ Integrationskonstante

bei $t = t_0$: $r = 0$, verschmelzen des Binärsystems

Zeit bis zum Verschmelzen: $\frac{5}{256}\frac{r^4}{\mu M^2} = t_0 - t =: \tau$

Frequenz:

$$E = -\frac{1}{2}\mu\omega^2 r^2 = -\frac{1}{2}\mu M^{2/3}\omega^{+2/3} = -\frac{1}{2} M_c^{5/3}\omega^{+2/3}\ , \quad \frac{dE}{dt} = -\frac{32}{5}(M_c\omega)^{10/3}$$

$$\leadsto -\frac{\omega^{-1/3}}{3} M_c^{5/3} \frac{d\omega}{dt} = -\frac{32}{5}(M_c\omega)^{10/3}$$

$$\omega^{-11/3}\frac{d\omega}{dt} = \frac{3}{5}\cdot 32 \cdot M_c^{5/3}$$

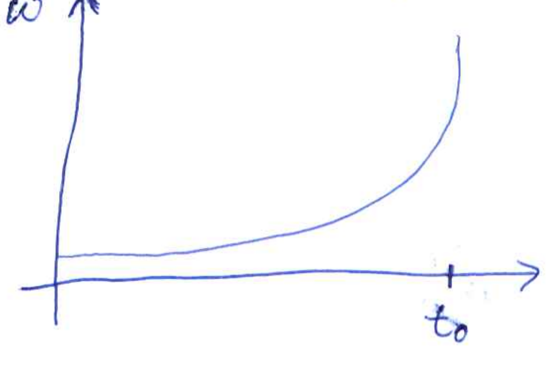
Integrieren:

$$-\frac{3}{8}\omega^{-8/3} = \frac{3}{5}\cdot 32\, M_c^{5/3}(t - t_0)$$

$$\omega^{-8/3} = +\frac{256}{5} M_c^{5/3} \underbrace{(t_0 - t)}_{\tau}$$

Frequenz der GW: $\omega_{gw} = 2\omega = 2\pi f_{gw}$

$$f_{gw} \simeq 134\,\text{Hz}\left(\frac{1.21\,M_\odot}{M_c}\right)^{5/8}\left(\frac{1s}{\tau}\right)^{3/8}$$



Frequenzverlauf legt M_c fest

GW 150914: $M_c \approx 30 M_\odot$

Korrekturen zu Newtonischer Gravitation (Periastrondrehung etc)

$\leadsto$ beide Massen m_1, m_2 können extrahiert werden