jan-steinhoff.de/lectures/tgwg2018/

Effective field theory (EFT) and gravitational waves (GW) from compact binary coalesence (CBC)

Why Effective Field Theory? (EFT)

pyramid of knowledge

$\Rightarrow$ abstractions, explanations } connections between branches of physics, e.g. oscillators, symmetries...

$\rightsquigarrow$ EFT an important part

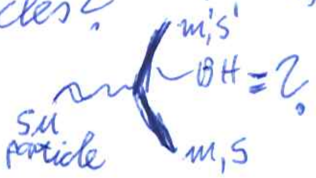
frontier connecting QFT and GR research: (double copy)

scattering (of black holes)

scattering angles/data $\longleftrightarrow$ scattering amplitudes (from "on-shell" methods)

black holes $\longleftrightarrow$ higher-spin massive particles?

(better way to treat spins?)

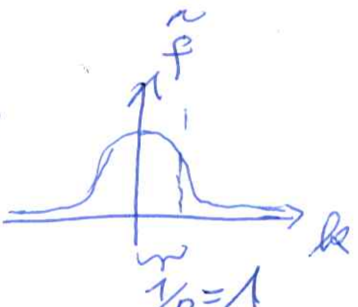


EFT $\rightsquigarrow$ coarse graining!

granular matter

average

$\sim\hat{=}$ Fourier trafo $\longleftrightarrow$



$$\langle\rho\rangle = \int d^3x'\, f(x')\,\rho(x-x')$$

$$\widetilde{\langle\rho\rangle} = \underbrace{\tilde{f}(k)}_{\text{low-pass filter}} \cdot \tilde{\rho}(k)$$

see e.g. derivation of macroscopic electrodynamics

applied to a star

$\xrightarrow[R\to 0,\ \Lambda\to\infty,\ \&\text{ rescaling},\ \tilde{f}(k)\simeq\text{const}\simeq 1]{\text{"continuum" limit}}$ m point particle

$$\widetilde{\langle\rho\rangle} = \tilde{f}(k)\cdot\tilde{\rho}(k) = \tilde{f}(k)\cdot\left(m + i k_i d^i - \tfrac{1}{2}k_i k_j Q^{ij} + \dots\right)$$

low pass filter, k must be small

Taylor series in k

comment on SM

in GR: from symmetries & matching

$$\langle\rho\rangle \underset{\tilde{f}(k)\simeq 1}{\approx} m\cdot\delta + d^i\partial_i\delta + \dots$$

but: cutoff $R\neq 0$, $\Lambda\neq\infty$ (physically)

$\rightsquigarrow$ renormalisation (even if theory nonsingular)

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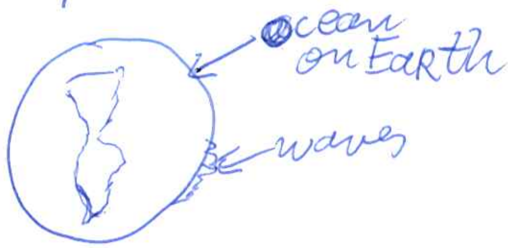
what are GWs ⇝ EFT

example: waves on oceans



which part of the water is the ocean and which one is the wave?

↳ approximate notion:
- small amplitude
- small wavelength / large k

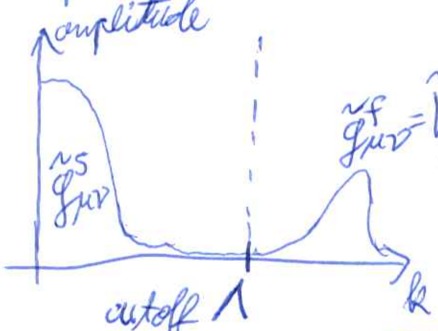
gravitational waves:

$$g_{\mu\nu} = \underbrace{g^s_{\mu\nu}}_{\langle g_{\mu\nu}\rangle} + \underbrace{g^f_{\mu\nu}}_{h_{\mu\nu}}$$

$\boxed{\begin{array}{l} s: \text{slow "modes"} \\ f: \text{fast "modes"} \end{array}}$

(not unique, problems with covariance; in practice, often obvious)

assumption:



$\Rightarrow \boxed{\tilde g^s \cdot \tilde h \approx 0}$

(approximate) decoupling ∇

(difference to linear theory)

$\boxed{\text{Separation of scales} \Leftrightarrow \text{decoupling}}$

(example: motion of moon)

field equations, schematically:

$$\underset{\substack{\uparrow \\ \text{curvature}}}{R} = \underset{\substack{\uparrow \\ \text{matter}}}{T} \longrightarrow R^s + h + h^2 + O(h^3) = T^s + T^f$$

fast modes ↗ $\Box h = T^f + O(h^2)$

slow modes ↘ $R^s = T^s + \langle h^2\rangle + O(h^3)$

$\langle h^2\rangle \rightsquigarrow T^{GW}_{\mu\nu}$

↑ projection

no $O(h)$ terms ⇝ decoupling ∇

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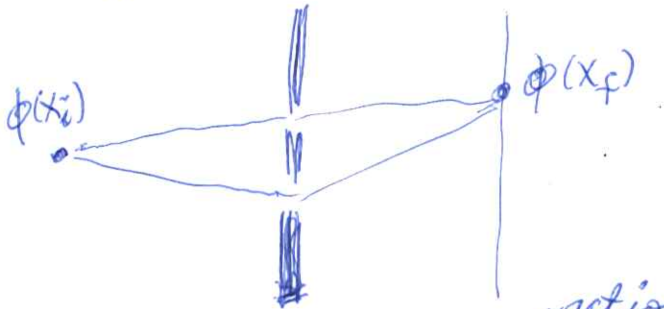
coarse graining ~~from~~ using path integral:

reminder:



observable eg amplitude $\langle i | f \rangle$ $= \underbrace{\int \mathcal{D}\phi}_{\text{integral over all paths / field configurations}} \, e^{iS}$ (something) ← action (here: ~~screen~~ double slit)

"$\mathcal{D}\phi = \prod_x d\phi(x)$"

now: $\phi = \phi^s + \phi^f$, integrate "out" over ϕ^f

$$\int \mathcal{D}\phi \, e^{iS} = \int \mathcal{D}\phi^s \underbrace{\int \mathcal{D}\phi^f e^{iS}}_{\text{write as } e^{iS_{eff}}}$$

$\sim$ $e^{iS_{eff}} = \int \mathcal{D}\phi^f e^{iS}$

S_{eff}: effective, coarse grained action
ϕ^f integrated out

classical part: drop quantum corrections to S_{eff}

for CBC:

mention:
spin, multipoles
tidal deformation
" heating
" resonances
tail effect ...

$\sim$ tower of EFTs (NB: start with QCD)

integrate out:
- body scale → point particle (from symmetries, matching)
- orbital " → weak-field, slow motion post-Newtonian approx. (PN) → exercise! (public code)
- wave " → waveform (observable)

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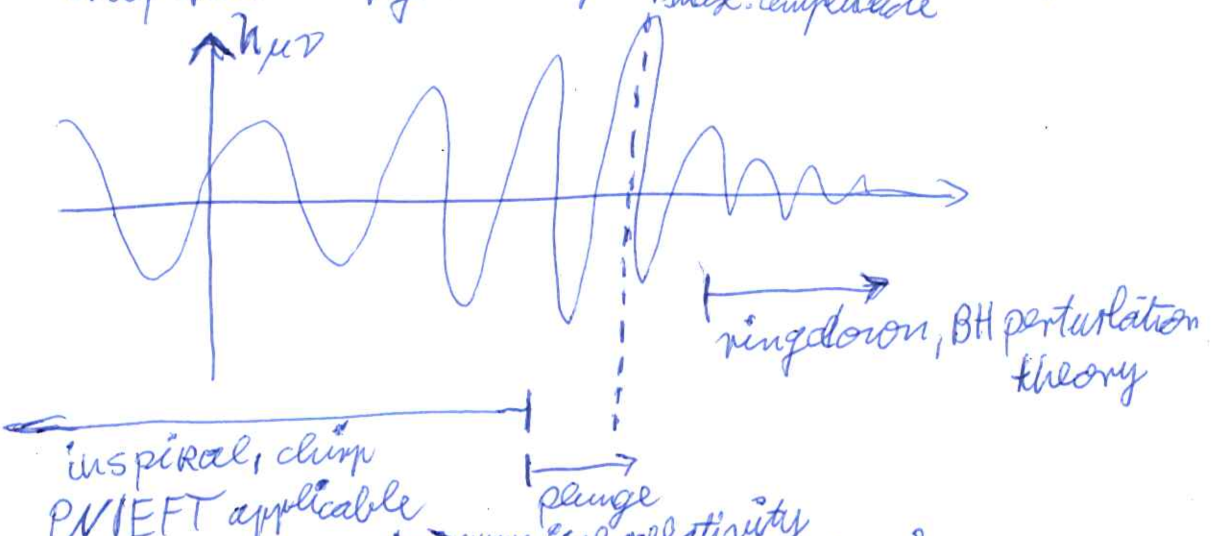
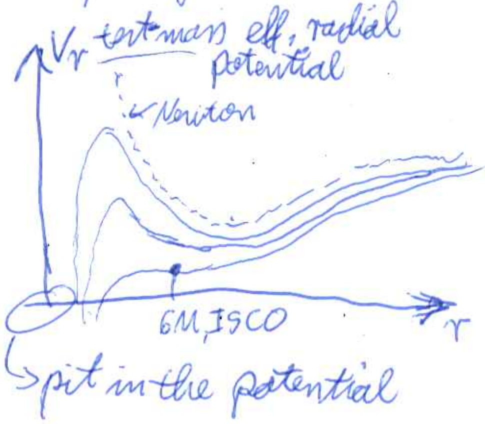
inspiral-merger-ringdown (IMR) waveforms (black holes)

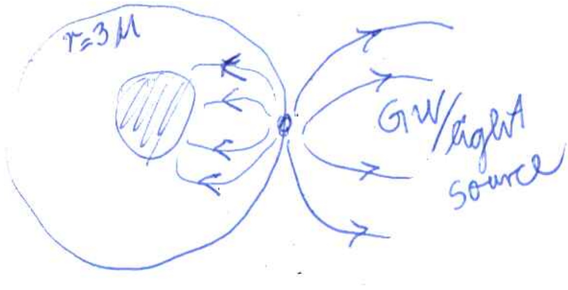


physics of IMR waveforms (nonspinning)

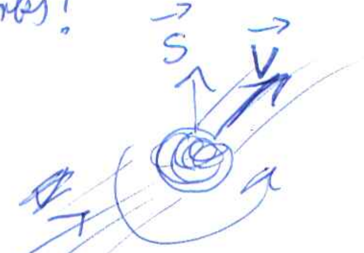
- early inspiral: orbit circularisation } amplitude & frequency grow (chirp)
- late inspiral ≈ circular
- innermost stable circular orbit (ISCO)
  gravity wins ⇝ plunge $r \sim 6M$
  (circular motion becomes unstable)



- light ring, $r \sim 3M$
  GW start to fall into the other BH
  ≈ half of the GW fall in
  ≈ point of max. amplitude



- common event horizon forms, $r \sim 2M$
  ↳ merger
- ringdown of final deformed BH
  GW resonate within light ring of final black hole
  frequency estimate (nonrot. BH):
  $$\omega_{rd} \approx 2\Omega \underset{\substack{\uparrow \\ \text{3rd Kepler}}}{\approx} \frac{2}{M}\left(\frac{M}{r}\right)^{3/2} \underset{\substack{\uparrow \\ r\approx 3M}}{\approx} \frac{2}{3\sqrt{3}\,M}$$ works!
- final (rapidly) spinning BH, recoil motion



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IMR waveform models

Phenomenological: emphasis on low comp. cost } not always a clear cut
effective-one-body: 〃 physics
(reduced order models: waveform compression & interpolation)

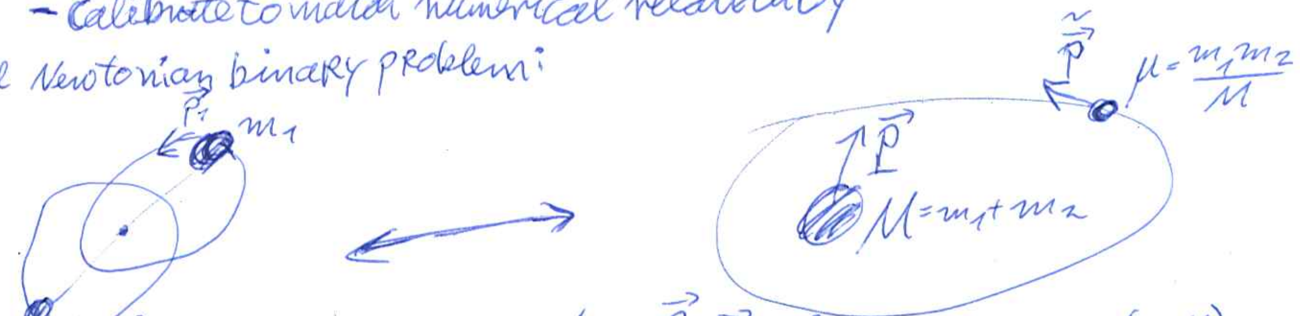
Effective-one-body (EOB) model

- combine PN results with test-mass motion → ISCO, light ring features
- attach ringdown (BH perturbation) at light ring

recent approach: backwards one body (BOB) for waveform from ISCO to end

- calibrate to match numerical relativity

recall Newtonian binary problem:



total lin. momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 = \text{const.}$
relative momentum $\tilde{\vec{p}} = \vec{p}_1 = -\vec{p}_2$ in center of mass frame (COM) $\vec{P} = 0$

relativistic:

$$S = \int d^4x \sqrt{-g} R + \int d\tau_1 \left[p_{1\mu} \frac{dx_1^\mu}{d\tau_1} - \lambda_1 \left(g^{\mu\nu}(x_1) p_{1\mu} p_{1\nu} + m_1^2 \right) \right] + (\text{body } 2)$$

λ_1: Lagrange multiplier

~~idea~~: mass-shell ~~constraint~~ $p_1^2 = -m_1^2$ encodes dynamics!

idea: map $p_{1,2}^2 = -m_{1,2}^2$ to mass-shells ~~constraints~~ for $P^\mu = p_1^\mu + p_2^\mu$, $\tilde{p}_\mu = ?$

assume $g_{\mu\nu} = \eta_{\mu\nu}$ for now

define $\tilde{p}_\mu \equiv \frac{1}{M} \begin{pmatrix} -p_{1\mu} p_2^\mu \\ P_0 \vec{p}_1 \end{pmatrix}$ in COM frame, $\vec{p}_1 = -\vec{p}_2$

then: $-\tilde{p}_\mu \tilde{p}^\mu = +\mu^2$

$$-P_\mu P^\mu = M_p^2 \equiv M^2 \left[1 + 2\nu \left(\frac{\tilde{p}_0}{\mu} - 1 \right) \right]$$

$\nu = \mu / M$ energy map $P_0 \leftrightarrow \tilde{p}_0$

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$H = P_0 =$ "real" Hamiltonian/energy
$H_e = \tilde{p}_0 =$ "effective" Hamiltonian/energy

$$H = \sqrt{M^2 \left[1 + 2\nu\left(\frac{H_e}{\mu} - 1\right)\right] + \vec{P}^2}$$ from kinematics, no interactions!

$\vec{P} = 0$: effective one body approximation (neglect recoil)

H_e for ~~co~~ grav. binary ?
- calculate PN result H_{PN}
- match $H = H_{PN}$ + canonical trafo

$\Rightarrow H_e$ up to "coordinate" freedom

result:
- 1PN: H_e ~~is~~ is test-mass in Schwarzschild
  $\Rightarrow$ combine PN & test-mass results
- 2PN : need to deform metric, only 1 additional term needed!
  $\Rightarrow$ very compact form of 2PN dynamics
- 3PN, 4PN : more complicated deformation of He
  still relatively compact