Tidal forces and mode resonances

in compact binaries

Sayan Chakrabarti[1] Térence Delsate[2] Jan Steinhoff[3]

[1] Indian Institute of Technology
Guwahati, India

[2] UMons
Mons, Belgium

[3] Instituto Superior Técnico
Lisbon, Portugal

Aveiro, October 9th, 2013

Supported by FCT (Portugal) and DFG (Germany)
through STE 2017 “Resonances of quasinormal modes and orbital motion in general relativistic compact binaries”

Important compact object: Neutron star

pics/neutronstar.pdf

Neutron star picture by D. Page
www.astroscu.unam.mx/neutrones/

„Lab“ for various areas in physics

- magnetic field, plasma
- crust (solid state)
- superfluidity
- superconductivity
- unknown matter in core
  (condensate of quarks, hyperons, kaons, pions, . . . ?)

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Tidal forces in binaries and resonances

tidal forces in inspiraling binaries $\longleftrightarrow$ oscillation modes of neutron stars

$\Rightarrow$ resonances!

resonances probably relevant for short gamma-ray bursts

[Tsang et.al., PRL **108** (2012) 011102]

Swift/BAT, nasa.gov

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binary neutron stars

mode spectrum from gravitational waves: gravito-spectroscopy

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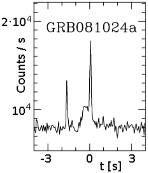

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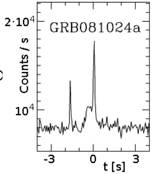

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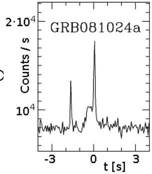

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Einstein Telescope

pics/spectral.p

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Tidal forces in Newtonian gravity

Simple case: linear perturbation of nonrotating barotropic stars

temperature-independent equation of state

see e.g. [Press, Teukolsky, ApJ **213** (1977) 183]

Displacement $\vec{\xi}$:= perturbed − unperturbed location of fluid elements

$$\ddot{\vec{\xi}} + \mathcal{D}\vec{\xi} = \text{(external forces)}$$

$$\mathcal{D}\vec{\xi} := -\vec{\nabla}\left\{\left[\frac{c_s^2}{\rho_0} + 4\pi G\Delta^{-1}\right]\vec{\nabla}\cdot(\rho_0\vec{\xi}\,)\right\}$$

ρ_0: unperturbed mass density c_s: speed of sound G: Newton constant

Properties of operator $\mathcal{D}$:

- contains differential operators $\vec{\nabla}$
- and also integral operator Δ^{-1}
- linear, nonlocal
- spherical symmetric
- Hermitian w.r.t. compact measure $dm_0 := \rho_0 d^3x$

  [Chandrasekhar, ApJ **139** (1964) 664–674]

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Normal modes (NM) in Newtonian gravity

Eigenfunctions of $\mathcal{D}$ are the normal oscillation modes of the star

$\mathcal{D}$ is Hermitian, compact integration measure $dm_0 := \rho_0 d^3x$

$\Rightarrow$ Discrete, real eigenvalues or oscillation frequencies $\omega_{n\ell}$

Eigenvalue equation:

$$\mathcal{D}\vec{\xi}^{\,\mathrm{NM}}_{n\ell m} = \omega^2_{n\ell}\vec{\xi}^{\,\mathrm{NM}}_{n\ell m}$$

Orthonormalization:

$$\int dm_0\, \vec{\xi}^{\,\mathrm{NM}\,\dagger}_{n'\ell' m}\, \vec{\xi}^{\,\mathrm{NM}}_{n\ell m} = \delta_{n'n}\delta_{\ell'\ell}\delta_{m'm}$$

Indices $\{\ell, m\}$ from spherical harmonics

Decomposition in terms of mode amplitudes $A_{n\ell m}(t)$

$$\vec{\xi} = \sum_{n\ell m} A_{n\ell m}(t)\vec{\xi}^{\,\mathrm{NM}}_{n\ell m}(\vec{x})$$

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Amplitude formulation

Displacement field $\vec{\xi}$ specified by

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and project onto orthonormal basis $\vec{\xi}^{\text{NM}}_{n\ell m}$

Result: uncoupled forced harmonic oscillators

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Radial/angular split of integral (spherical harmonics)

$I_{n\ell} \sim$ radial integration part of $f_{n\ell m}$

overlap integral

$$f_{n\ell m} \propto I_{n\ell} \times (\ell\text{-pole of } \Phi_{\mathrm{ext}})$$

Overlap integrals $I_{n\ell}$

- Coupling constants between modes to external field
- Determine energy exchange between orbital motion and modes
- Together with frequencies $\omega_{n\ell}$ they define the gravito-spectrum (measurable through gravitational waves)

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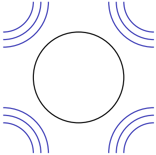
Different perspective on the Newtonian theory

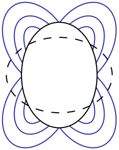
Chakrabarti, Delsate, Steinhoff, arXiv:1306.5820

Example: quadrupolar sector $\ell = 2$



linear response

$$\xrightarrow{F}$$



external quadrupolar field $\longrightarrow$ deformation $\longrightarrow$ quadrupolar response

quadrupolar response:

$$F = \sum_n \frac{I_{n\ell}^2}{\omega_{n\ell}^2 - \omega^2}$$

$\omega_{n\ell}$: mode frequency
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R: radius

poles $\Rightarrow$ resonances!

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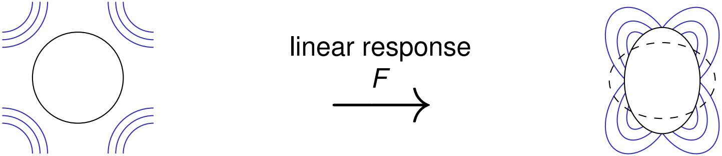

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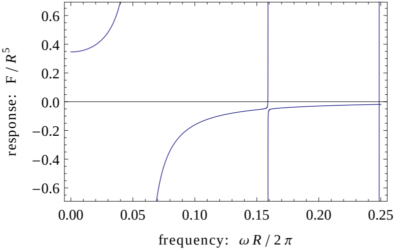

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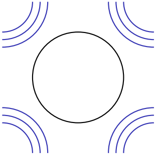

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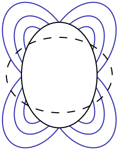

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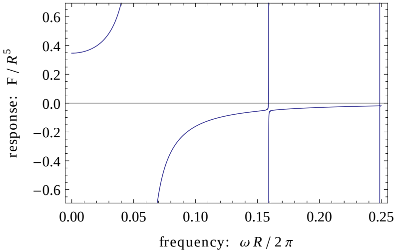

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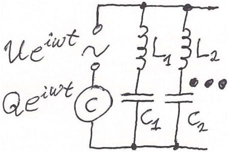
Tacoma Bridge

Analogy with electronics



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Q: quadrupole
E: external tidal field

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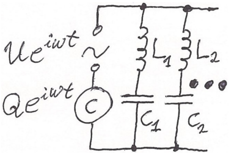

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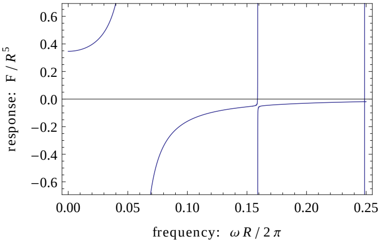

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Philosophy of the approach

Starting point: single object, e.g., neutron star

Idea

Multipoles describe compact object on macroscopic scale

$\longleftrightarrow$

state variables (p, V, T)	$\longleftrightarrow$	multipoles (M, S, Q)
equations of state	$\longleftrightarrow$	effective action
correlation	$\longleftrightarrow$	response

Approximations for binary system using effective theory:

- Effective point-particle action with dynamical multipoles
- Response functions (propagators) for multipoles

$\Rightarrow$ Predictions: gravitational waves, gamma-ray bursts, pulsar timing

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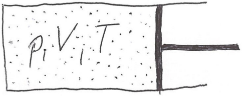

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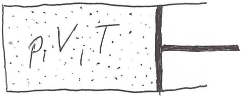

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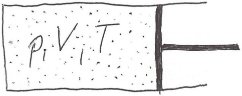

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Intermediate conclusion

Newtonian theory in terms of effective action and response functions:

- Alternative computation of overlap integral through fit of F
- Immediately generalizes to more complicated situations
- Can be generalized to the relativistic case

Relativistic tidal interactions

Status: expansion around adiabatic case

$$F(\omega) = 2\mu_2 + i\omega\lambda + 2\omega^2\mu_2' + \mathcal{O}(\omega^3)$$

- μ_2: 2nd kind relativistic Love number [Hinderer, ApJ **677** (2008) 1216; Damour, Nagar, PRD 80 (2009) 084035; Binnington, Poisson, PRD 80 (2009) 084018]
- λ: absorption [Goldberger, Rothstein, PRD **73** (2006) 104030]
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Motivation:

- Adiabatic tidal effects may not be sufficient [Maselli, Gualtieri, Pannarale, Ferrari, PRD **86** (2012) 044032]
- Definition of relativistic overlap integrals
- Resonances between oscillation modes and orbital motion:
  - Numerical simulations of binary neutron stars for eccentric orbits [Gold, Bernuzzi, Thierfelder, Brügmann, Pretorius, PRD **86** (2012) 121501]
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Relativistic case: generic problems

- Analog to Hermitian operator $\mathcal{D}$ not available in the relativistic case
- Substitute neutron star by point particle reproducing large-scale field

- Need to handle an inhomogeneous Regge-Wheeler equation with effective point-particle source S representing a neutron star

$$\frac{d^2X}{dr_*^2} + \left[\left(1 - \frac{2M}{r}\right)\frac{\ell(\ell+1) - \frac{6M}{r}}{r^2} + \omega^2\right] X = S$$

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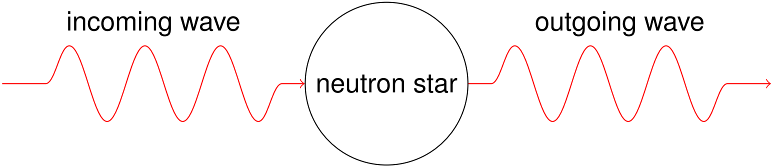

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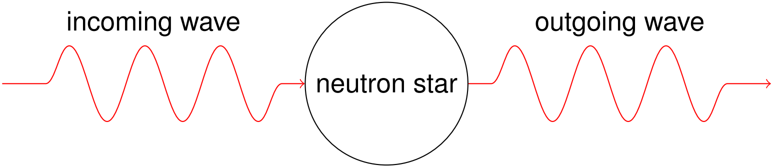

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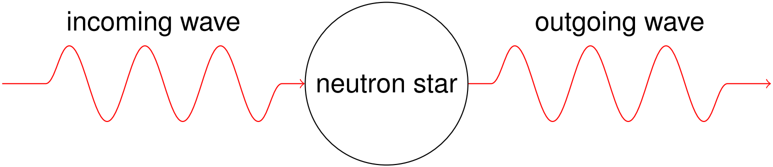
$$\frac{d^2X}{dr_*^2} + \left[\left(1 - \frac{2M}{r}\right)\frac{\ell(\ell+1) - \frac{6M}{r}}{r^2} + \omega^2\right] X = S$$

- How to construct solutions corresponding to external field and response?

Relativistic case: generic problems

- Analog to Hermitian operator $\mathcal{D}$ not available in the relativistic case
- Substitute neutron star by point particle reproducing large-scale field



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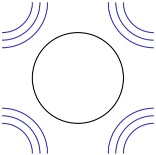
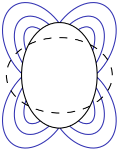
Identification of external field and response



external quadrupolar field



quadrupolar response

Newtonian:

$r^{\ell+1}$ $\qquad$ $r^{-\ell}$

adiabatic $\omega = 0$:

$r^{\ell+1}\ {}_2F_1(\dots; 2M/r)$ $\qquad$ $r^{-\ell}\ {}_2F_1(\dots; 2M/r)$

relativistic:

$X^{\ell}_{\rm MST}$ $\qquad$ $X^{-\ell-1}_{\rm MST}$

where [Mano, Suzuki, Takasugi, PTP **96** (1996) 549]

$$X^{\ell}_{\rm MST} = e^{-i\omega r}(\omega r)^{\nu}\left(1-\frac{2M}{r}\right)^{-i2M\omega}\sum_{n=-\infty}^{\infty}\dots\times\left[\frac{r}{2M}\right]^{n}{}_2F_1(\dots; 2M/r)$$

Renormalized angular momentum, transcendental number: $\nu = \nu(\ell, M\omega)$

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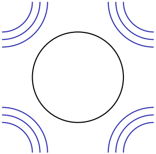

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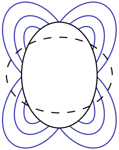

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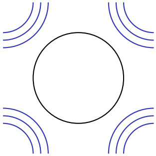

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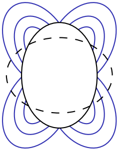

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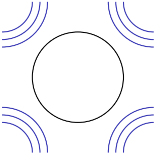

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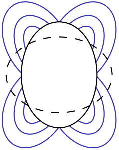

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Identification of external field and response by
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Relativistic response

Chakrabarti, Delsate, Steinhoff, arXiv:1304.2228

- Numerical neutron star perturbation matched to
$$X = A_1 X^{\ell}_{\mathrm{MST}} + A_2 X^{-\ell-1}_{\mathrm{MST}}$$
- X^{ℓ}_{MST}, $X^{-\ell-1}_{\mathrm{MST}}$ related to effective point-particle source via
  variation of parameters
- Point-particle source requires regularization (here: Riesz-kernel)
- Regularization introduces dependence on scale μ_0

- Fit for the response:

$$F(\omega) \approx \sum_n \frac{I_n^2}{\omega_n^2 - \omega^2}$$

- Just like in Newtonian case!
- Relative error less than 2%
- Relativistic overlap integrals: I_n
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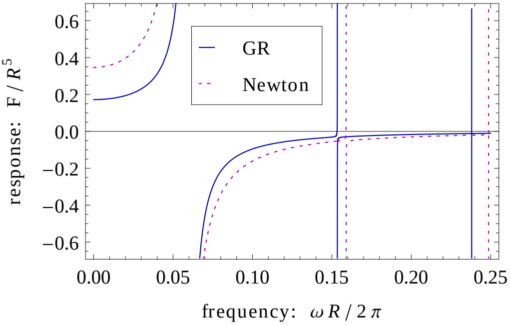
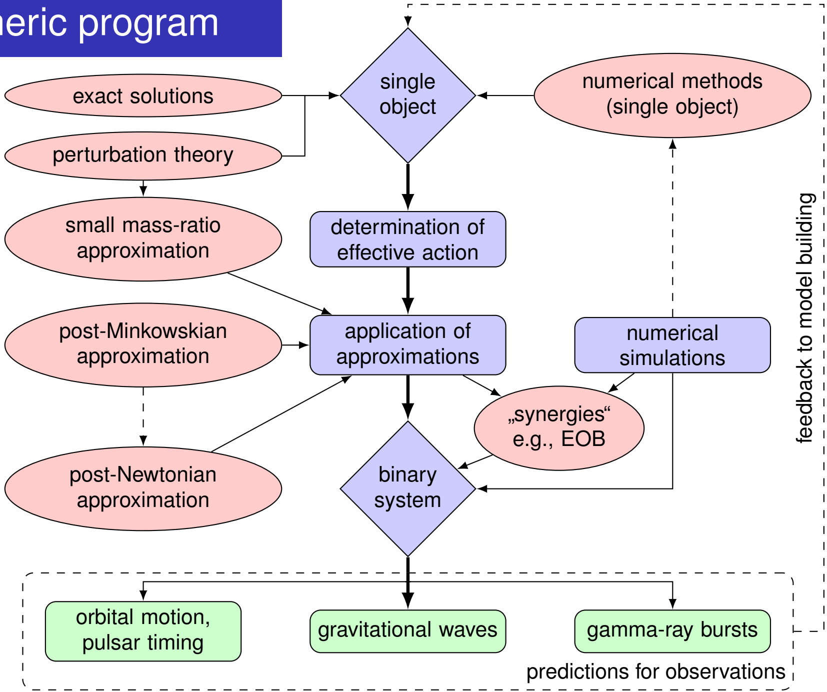


Generic program



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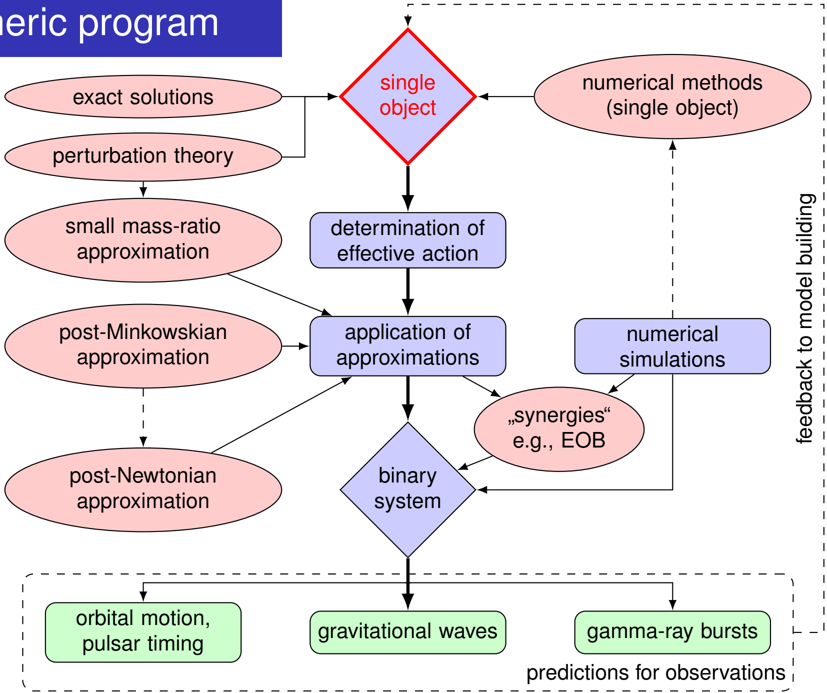

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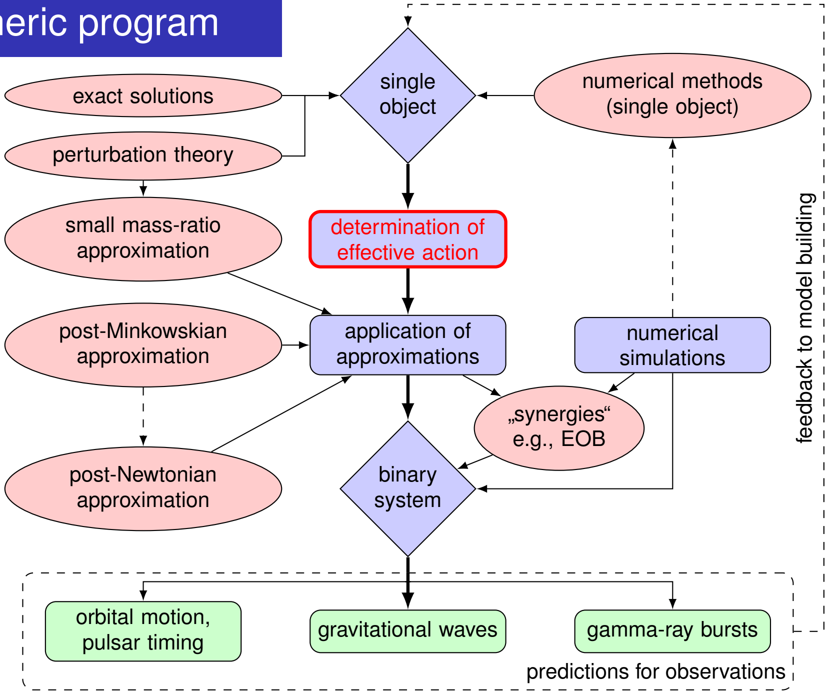

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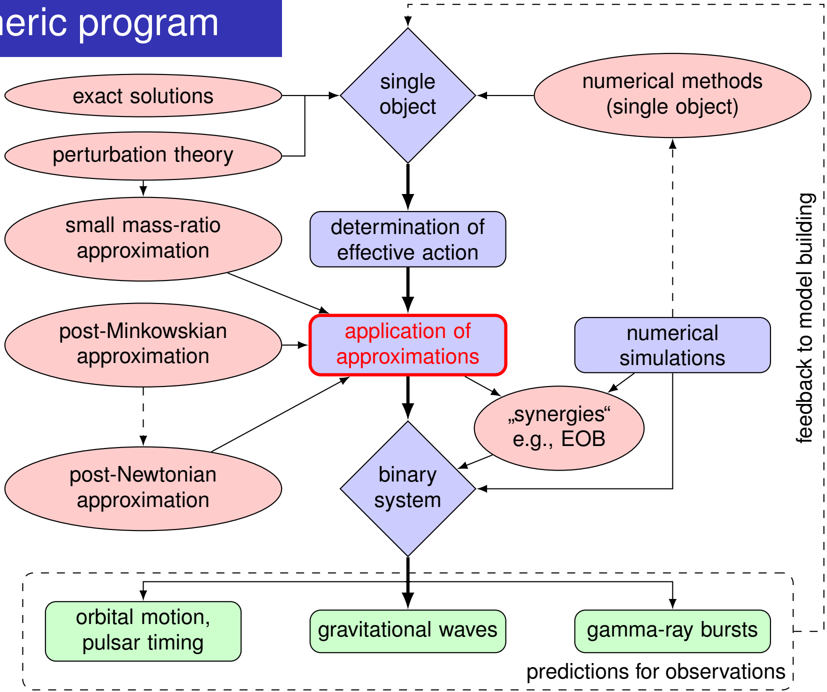

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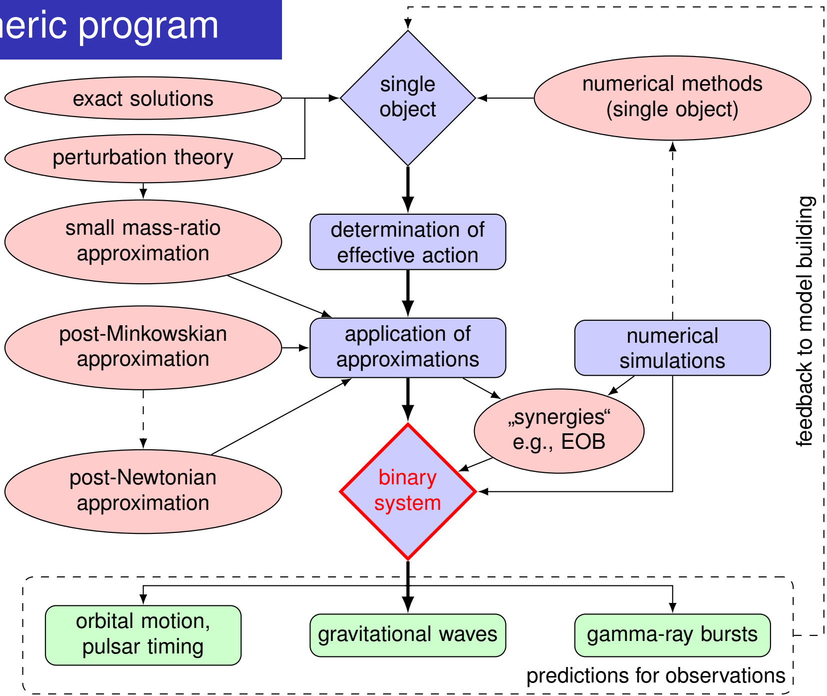

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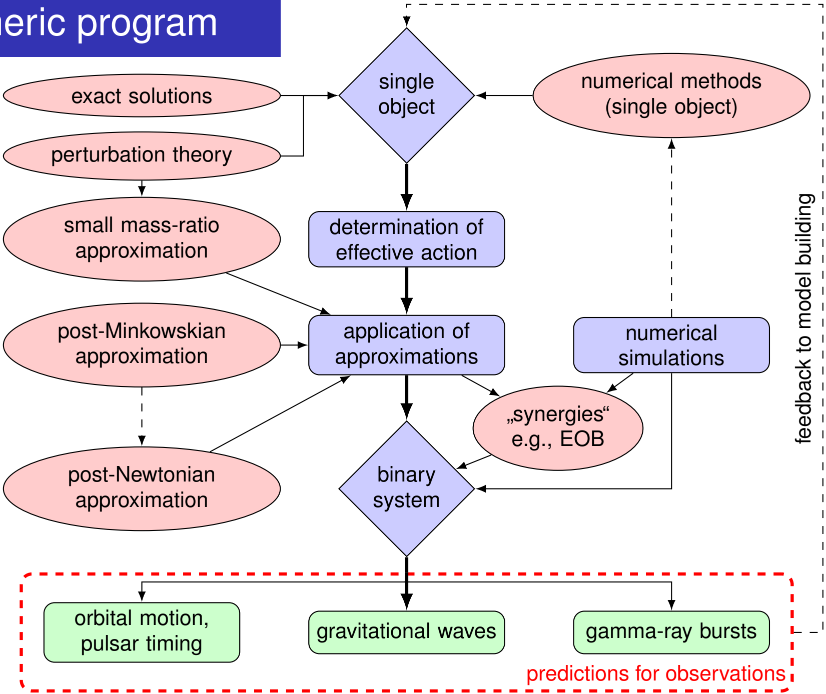

Conclusions and outlook

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- We defined the relativistic quadrupolar response
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- Response is completely analogous to Newtonian case
- We defined relativistic overlap integrals
- Important step towards gravito-spectroscopy using gravitational waves

Outlook:

- More realistic neutron star models:
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- Connection to gamma-ray bursts:
  shattering of crust, instabilities of modes
- Dimensional regularization
- Other multipoles
  based on action in [Goldberger, Ross, PRD **81** (2010) 124015]
- 2nd Love number of rotating black holes

Thank you for your attention

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