Spin$^\infty$

Recent analytic results for spin effects of black hole binaries

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Spin ⇒ Twisted Spacetime

pics/GPB

angular momentum:

- gravito-magnetic field
- dragging of reference frames

measured by Gravity Probe B →

← the second event: GW151226
likely contains spin effects

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pics/GW151226spins

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What is our concern about waveform models

for spinning binary black holes (BBHs)?

post-Newtonian (PN) approximation: expansion around $\frac{1}{c} \to 0$ (Newton)

Structure of the binding energy e as a function of $x \sim \omega^{2/3}$ at nPN order:

- nonspinning: $e(x) \sim x^{n+1} f_{\rm NS}(m_1, m_2)$
- spin-orbit: $e(x) \sim x^{n+1} f_{\rm SO}(m_1, m_2)\, \vec{S}_1 \cdot \vec{L}$
- higher orders in spin: $e(x) \sim$ polynomial in
  $\vec{S}_i \cdot \vec{L}, \quad \vec{S}_i \cdot \vec{n}, \quad \vec{S}_i \cdot (\vec{n} \times \vec{L})$
  Important for the strong-field regime!

Calibration to numerical relativity in the last case?
$\Rightarrow$ probably a whack-a-mole game!
(too many free functions to calibrate)

But: higher order spin effects are rather simple to calculate analytically

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Results in post-Newtonian approximation with spin

conservative part of the motion of the binary

post-Newtonian (PN) approximation: expansion around $\frac{1}{c} \to 0$ (Newton)

order	c^0 N	c^{-1}	c^{-2} 1PN	c^{-3}	c^{-4} 2PN	c^{-5}	c^{-6} 3PN	c^{-7}	c^{-8} 4PN
non spin	✔		✔		✔		✔		✔
spin-orbit				✔		✔		✔	
S_1^2					✔		✔		✔
$S_1 S_2$					✔		✔		✔
Spin^3								✔ (✓)	
Spin^4									✔ (✓)
⋮									⋱

✔ known (✓) partial ✔ derived 2015

Work by many people ("just" for the spin sector): Barker, Blanchet, Bohé, Buonanno, O'Connell, Damour, D'Eath, Faye, Hartle, Hartung, Hergt, Jaranowski, Marsat, Levi, Ohashi, Owen, Perrodin, Poisson, Porter, Porto, Rothstein, Schäfer, Steinhoff, Tagoshi, Thorne, Tulczyjew, Vaidya

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What is spin$^\infty$?

New approach: compute diagonally!

✓ known (✓) partial ✓ derived 2015

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Summing spin to infinity (leading PN order)

Started with a conjecture from looking at S^2, S^3, and S^4.
An effective action computation leads to:

$$H_{\rm LO}^{\rm BBH} = \frac{\vec{P}^2}{2\mu} - \mu U + 4\vec{P}\cdot\vec{A} + \frac{1}{2}\vec{P}\times\left[\frac{\vec{S}_1}{m_1^2} + \frac{\vec{S}_2}{m_2^2}\right]\cdot\vec{\nabla}\mu U$$

$$U = \cos(\vec{a}_0\cdot\vec{\nabla})\frac{M}{R}, \qquad \vec{A} = -\frac{1}{2}\vec{a}_0\times\vec{\nabla}\frac{\sin(\vec{a}_0\cdot\vec{\nabla})}{\vec{a}_0\cdot\vec{\nabla}}\frac{M}{R}$$

where $M = m_1 + m_2$, $\quad \mu = M_1 m_2/M$,
$\vec{a}_0 = \vec{a}_1 + \vec{a}_2$, $\quad \vec{a}_i = \vec{S}_i/m_i$

Trigonometric functions due to:
(mass l-pole) $+\, i$ (current l-pole) $= m(ia)^l$

Closed form in oblate spheroidal coord.:

$$U = \frac{Mr}{r^2 + a_0^2\cos^2\theta}, \qquad \vec{A} = -\frac{U}{2}\frac{\vec{R}\times\vec{a}_0}{r^2 + a_0^2}$$

linearized harmonic-gauge Kerr metric!

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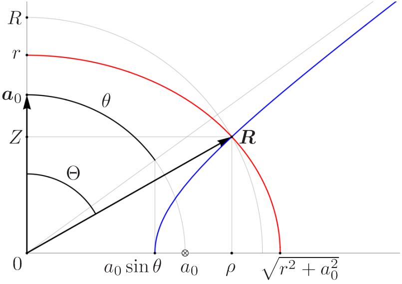

oblate-spheroidal coord. r, θ

Interpretation of the result

Spin interactions to leading PN order:

$$\sum_{l_1}\sum_{l_2}(l_1\text{-pole}) \times (l_2\text{-pole}) \sim \text{mass moving in a spacetime with Kerr radius } a_0$$

where $a_0 = $ (Kerr radius of BH 1) $+$ (Kerr radius of BH 2)

radii of the ring singularities add

Similar: in Newtonian gravity, the binary motion corresponds to a motion of a reduced mass μ in the field of the total mass $M = m_1 + m_2$.

$\Rightarrow$ Important for the Effective-One-Body (EOB) waveform model

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Conclusions

spin$^\infty$:

- not an approximation with well defined purpose
- but the result gives insights into the structure spin interactions of BBH!

  "compute-what-you-can-and-look-for-structure" approximation
- insight: relations between test-body and binary motion.
- very interesting for improving/building an EOB(-like) waveform model.

to be explored:

- approximation schemes other than PN
- connection to Newman-Janis algorithm
- connection to higher-spin field

One should. . .

calculate interactions that are hard to extract from numerical simulations
look for structure and resummations

Thank you for your attention

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