Tidal forces and mode resonances

in compact binaries

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Outline

Strong gravity and compact objects

General relativity is expected to break down for strong field strengths.

Currently our best chance to test strong gravity:

compact astrophysical objects!

Black holes: clean (vacuum solutions); but observations difficult

Neutron stars (NS):

- strong gravity in NS interior
- alternative theories can predict drastically different NS structure: oscillation modes, mass-radius relation, spontaneous scalarization

but partly unknown matter properties . . .

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A neutron star model

pics/neutronstar

Neutron star picture by D. Page
www.astroscu.unam.mx/neutrones/

„Lab“ for various areas in physics

- magnetic field, plasma
- crust (solid state)
- superfluidity
- superconductivity
- unknown matter in core
  condensate of quarks, hyperons, kaons, pions, . . . ?

  accumulation of dark matter ?

Related objects:

- pulsars
- magnetars
- quark stars

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Tidal forces in binaries and resonances

tidal forces in inspiraling binaries $\longleftrightarrow$ oscillation modes of neutron stars

$\Rightarrow$ resonances!

resonances probably relevant for short gamma-ray bursts

[Tsang et.al., PRL **108** (2012) 011102]

pics/doublepulsar

binary neutron stars

Swift/BAT, nasa.gov

mode spectrum from gravitational waves: gravito-spectroscopy

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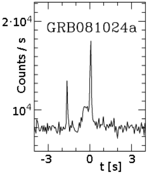

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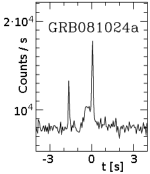

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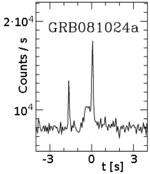

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pics/et

Einstein Telescope

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Effective theory for compact objects

See: [Goldberger, Rothstein, PRD **73** (2006) 104029]

[Goldberger, Ross, PRD **81** (2010) 124015]

Effective theory: important idea from statistical physics, quantum field theory

Starting point: An action for a "full theory" S_{full}

"Integrate out" short scale (UV) part of the metric g_{UV}:

$$\exp\left(\tfrac{i}{\hbar} S_{\text{eff}}[g_{IR}, \text{matter}]\right) = \int Dg_{UV} \, \exp\left(\tfrac{i}{\hbar} S_{\text{full}}[g_{IR} + g_{UV}, \text{matter}]\right)$$

We are only interested in the classical part of the path integral.

On large scales (IR), a mass distribution looks like a point-particle:

$$S_{\text{eff,matter}}[g_{IR}, \text{matter}] = \int d\tau \left[-m - \frac{1}{2} E_{ab} Q^{ab} + \dots \right]$$

$$E_{ab} \sim R_{acbd} u^c u^d, \qquad Q^{ab} \sim \text{mass quadrupole}$$

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Philosophy of the approach

Starting point: single object, e.g., neutron star

Idea

Multipoles describe compact object on macroscopic scale

$\longleftrightarrow$

state variables (p, V, T)	$\longleftrightarrow$	multipoles (m, S , Q)
equations of state	$\longleftrightarrow$	effective action
correlation	$\longleftrightarrow$	response

Approximations for binary system using effective theory:

- Effective point-particle action with dynamical multipoles
- Response functions (propagators) for multipoles

$\Rightarrow$ Predictions: gravitational waves, gamma-ray bursts, pulsar timing

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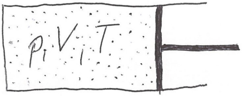

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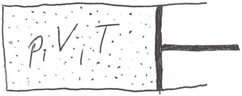

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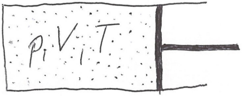

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Example: adiabatic tidal deformation

for NS, e.g. Hinderer & Flanagan (2008); Damour, Nagar (2009); Binnington, Poisson (2009)

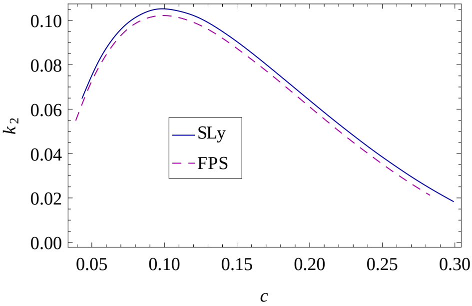
- Static, linear NS perturbation:

$$-Q = \mu_2 E$$

- Tidal force E (curvature)
- Dim.-less 2nd Love number k_2:

$$k_2 = \frac{3}{2}\frac{\mu_2}{R^5}$$

- Measure for grav. polarizability
- Compactness $c = \frac{Gm}{R}$



see Damour, Nagar, PRD **80** (2009) 084035

- k_2, μ_2 encode strong gravity effects in the interior
- beyond the adiabatic case:

  [Bini, Damour, Faye, PRD **85** (2012) 124034]

  [Maselli, Gualtieri, Pannarale, Ferrari, PRD **86** (2012) 044032]

Outline

Tidal forces in Newtonian gravity

Simple case: linear perturbation of nonrotating barotropic stars

temperature-independent equation of state

see e.g. [Press, Teukolsky, ApJ **213** (1977) 183]

Displacement $\vec{\xi}$:= perturbed − unperturbed location of fluid elements

$$\ddot{\vec{\xi}} + \mathcal{D}\vec{\xi} = \text{(external forces)}$$

$$\mathcal{D}\vec{\xi} := -\vec{\nabla}\left\{\left[\frac{c_s^2}{\rho_0} + 4\pi G\Delta^{-1}\right]\vec{\nabla}\cdot(\rho_0\vec{\xi}\,)\right\}$$

ρ_0: unperturbed mass density c_s: speed of sound G: Newton constant

The operator $\mathcal{D}$:

- differential $\vec{\nabla}$ and integral Δ^{-1} operators
- linear, nonlocal, spherical symmetric
- Hermitian w.r.t. compact measure $dm_0 := \rho_0 d^3x$

  [Chandrasekhar, ApJ **139** (1964) 664–674]

⇒ Eigenfunctions of $\mathcal{D}$ are the normal oscillation modes of the star with discrete, real eigenvalues $\omega_{n\ell}^2$ → oscillation frequencies

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Effective theory point of view on tidal interactions
Chakrabarti, Delsate, Steinhoff, PRD **88** (2013) 084038

Full theory: variational fluid dynamics, linear perturbation

Integrate out small scales, mode decomposition:

$$\vec{\xi} = \sum_{n\ell m} A_{n\ell m}(t) \vec{\xi}^{\text{NM}}_{n\ell m}(\vec{x})$$

Result for effective action, quadrupolar truncation $\ell = 2$:

$$S_{\text{eff,matter}} = \int dt \left[\frac{1}{2} m \dot{\vec{z}}^2 - m\Phi + \frac{1}{2} \sum_{n,m} \left[|\dot{A}_{n2m}|^2 - \omega_n^2 |A_{n2m}|^2 - I_n A_{n2m} E_{2m} \right] + \ldots \right]$$

$$E_{2m} \sim E_{ab} = \partial_a \partial_b \Phi, \qquad n = \text{mode number}$$

Overlap integrals I_n: coupling constants
Gravito-spectrum: ω_n and I_n

Try this with a nonlinear extension of Newtonian gravity?

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Convenient concept: response function

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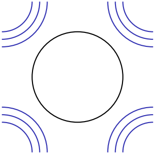
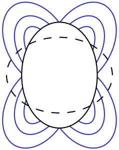


linear response

$\xrightarrow{F}$



external quadrupolar field $\longrightarrow$ deformation $\longrightarrow$ quadrupolar response

$\Phi \sim r^{\ell}$ $\qquad$ $\Phi \sim r^{-\ell-1}$

quadrupolar response:

$$F = \sum_n \frac{I_n^2}{\omega_n^2 - \omega^2}$$

ω_n: mode frequency
I_n: overlap integral
R: radius

poles $\Rightarrow$ resonances!

Computation of I_n through fit of F $\Rightarrow$ generalizes to relativistic case

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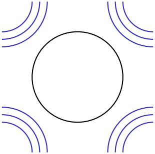

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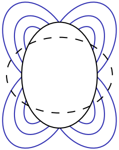

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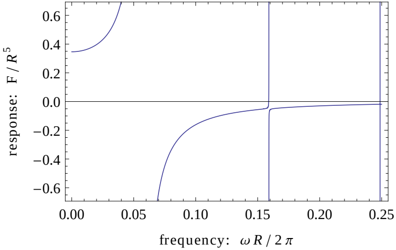

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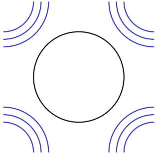

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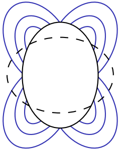

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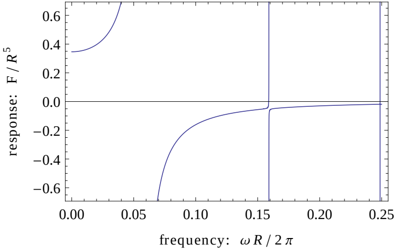

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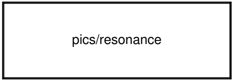

Tacoma Bridge

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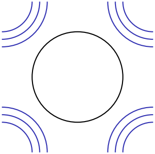

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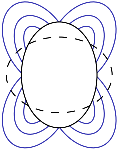

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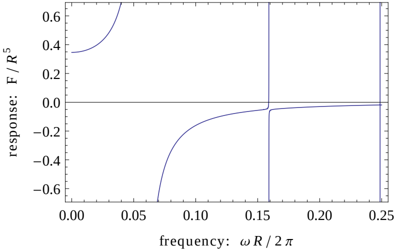

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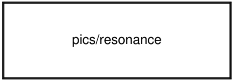

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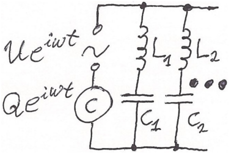
Analogy with electronics



$$\frac{Q}{U} = \frac{1}{i\omega\, Z}$$

$$= \sum_n \frac{\frac{1}{L_n}}{\frac{1}{C_n L_n} - \omega^2}$$

$$\frac{Q}{E} =: F$$

$$= \sum_n \frac{I_{n\ell}^2}{\omega_{n\ell}^2 - \omega^2}$$

Q: quadrupole
E: external tidal field

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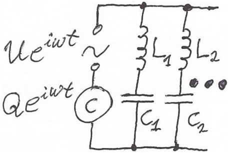

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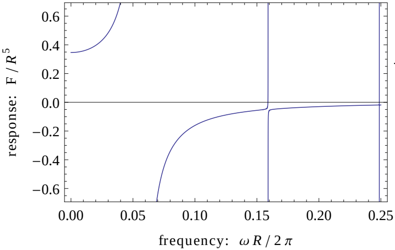

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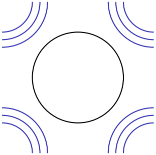
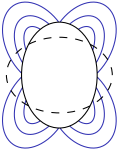
Outline

Identification of external field and response



external quadrupolar field



quadrupolar response

Newtonian:

$r^{\ell+1}$ $\quad\quad$ $r^{-\ell}$

relativistic, adiabatic $\omega = 0$:

$r^{\ell+1}\ {}_2F_1(\ldots; 2m/r)$ $\quad\quad$ $r^{-\ell}\ {}_2F_1(\ldots; 2m/r)$

relativistic, generic ω:

X^{ℓ}_{MST} $\quad\quad$ $X^{-\ell-1}_{\mathrm{MST}}$

where [Mano, Suzuki, Takasugi, PTP **96** (1996) 549]

$$X^{\ell}_{\mathrm{MST}} = e^{-i\omega r}(\omega r)^{\nu}\left(1 - \frac{2m}{r}\right)^{-i2m\omega} \sum_{n=-\infty}^{\infty} \cdots \times \left[\frac{r}{2m}\right]^{n} {}_2F_1(\ldots; 2m/r)$$

Renormalized angular momentum, transcendental number: $\nu = \nu(\ell, m\omega)$

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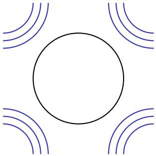

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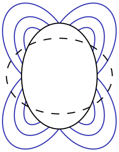

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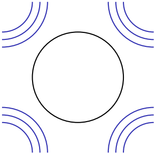

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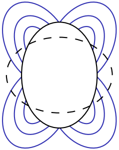

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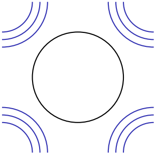

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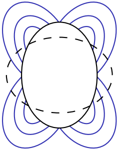

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Identification of external field and response by considering generic ℓ (analytic continuation)

Relativistic response

Chakrabarti, Delsate, Steinhoff, arXiv:1304.2228

- Numerical neutron star perturbation matched to

$$X = A_1 X^{\ell}_{\text{MST}} + A_2 X^{-\ell-1}_{\text{MST}} \quad \text{(homogeneous solution)}$$

- X^{ℓ}_{MST}, $X^{-\ell-1}_{\text{MST}}$ related to effective point-particle source via

  variation of parameters (inhomogeneous solution)

- Point-particle source requires regularization (here: Riesz-kernel)
- Regularization introduces dependence on scale μ_0

- Fit for the response:

$$F(\omega) \approx \sum_n \frac{I_n^2}{\omega_n^2 - \omega^2}$$

- Just like in Newtonian case!
- Relative error less than 2%
- Relativistic overlap integrals: I_n
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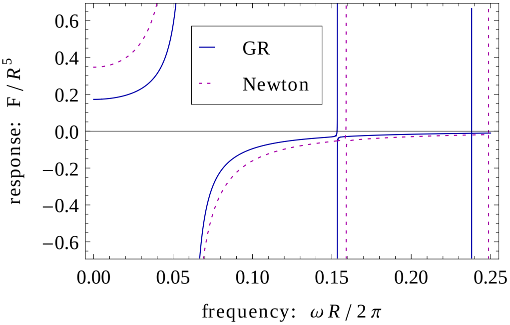

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Next steps:

- More realistic neutron star models: rotation, crust, . . .
- Connection to gamma-ray bursts: shattering of crust

The search for universality among different NS models:

- I-Love-Q [Yagi, Yunes, Science **341** (2013) 365]
- strong tides [Maselli, Cardoso, Ferrari, Gualtieri, Pani, PRD **88** (2013) 023007]
- I-Q, strong rotation [Doneva, Yazadjiev, Stergioulas, Kokkotas, `arXiv:1310.7436`; Chakrabarti, Delsate, Gürlebeck, Steinhoff, `arXiv:1311.6509`]
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