Influence of internal structure on the motion

of test bodies in extreme mass ratio situations

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13th Marcel Grossmann Meeting, 2012, Stockholm

DFG : STE 2017/1-1[1] and LA-905/8-1[2]

EOM for test-bodies with internal structure

$$\frac{\delta p_a}{ds} = 0 + \frac{1}{2} R_{abcd} u^b S^{cd} + \frac{1}{6} \nabla_a R_{bcde} J^{bcde} + \ldots$$

$$\frac{\delta S^{ab}}{ds} = 2p^{[a} u^{b]} - \frac{4}{3} R^{[a}{}_{cde} J^{b]cde} + \ldots$$

$$\frac{\delta J^{abcd}}{ds} = ???$$

- Geodesic equation: momentum p_μ
- Mathisson (1937), Papapetrou (1951): spin / dipole S^{ab}
- Dixon (∼1974): quadrupole J^{abcd}, ...
- EOM for p_a and S^{ab} follow from theory! $T^{ab}{}_{;b} = 0 \rightsquigarrow$ EOM

Conserved Quantities:

- For a Killing vector field ξ^a: $E_\xi = p_a \xi^a + \frac{1}{2} S^{ab} \nabla_a \xi_b$
- Neglecting J^{abcd} etc.: mass $\underline{m} := \sqrt{-p_a p^a}$ or $m := u^a p_a$ (SSC dep.)
  spin-length $S = \sqrt{\frac{1}{2} S_{ab} S^{ab}}$

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Effective potential for Kerr background

We want to solve for p^a and S^{ab} (10 DOF) in terms of

$$E := E_{\partial_t}, \quad J := E_{-\partial_\theta}, \quad \underline{m}, \quad S, \quad M, \quad a, \quad \text{and coords.}$$

We need 10 (independent) equations:

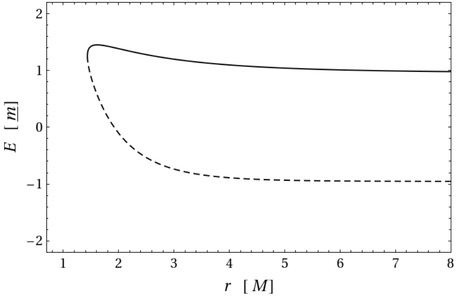
- Definitions of $\{E, J, \underline{m}, S\}$ $\rightsquigarrow$ 4 eqs.
- Spin supplementary condition (SSC): $S^{ab}p_b = 0$ $\rightsquigarrow$ 3 indep. eqs.
- Equatorial orbits ($\theta = \pi/2$): $p^\theta = 0$ $\rightsquigarrow$ 1 eq.
- Aligned spin: $S^{a\theta} = 0$ $\rightsquigarrow$ 2 indep. eqs.

Solution for p^r:

$$(p^r)^2 = \alpha E^2 + \beta E + \gamma$$

- We must have $(p^r)^2 \geq 0$
- roots of $(p^r)^2 = 0$ important:
  - turning points
  - circular orbits

circular orbit $\rightsquigarrow$ minimum $\rightsquigarrow r$



roots of $(p^r)^2 = 0$ for $J = 4M\underline{m}$, $S = 0$, $a = +0.9$

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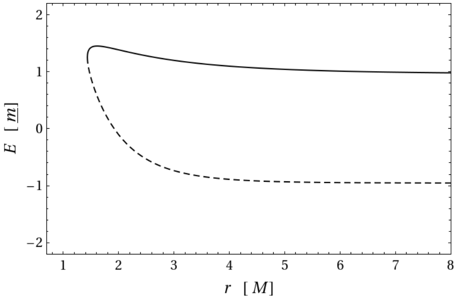

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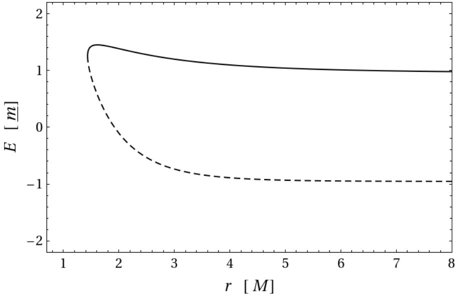

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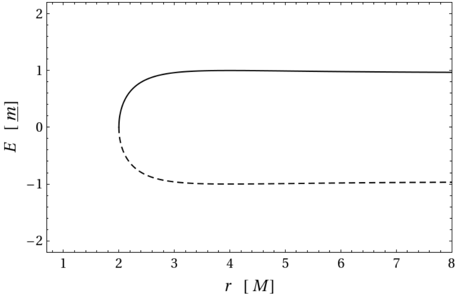

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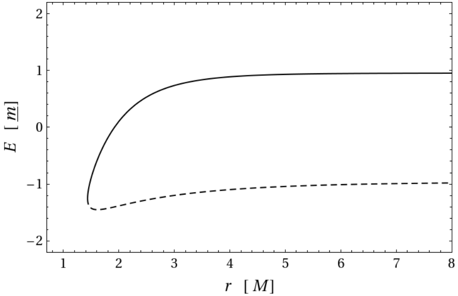

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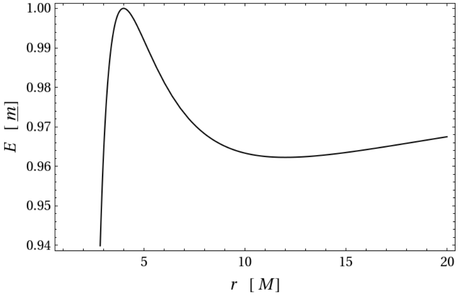

upper root of $(p^r)^2 = 0$ for $J = 4M\underline{m}$, $S = 0$, $a = 0$

The quadrupole model inspired by EFT/PN

see e.g. arXiv:0804.0260, hep-th/0511133, arXiv:0911.5041

$$R_M = \mu u - \underbrace{\frac{1}{\mu u} B_{ab} S^a u_c S^{cb}}_{\text{SSC preserving}} - \underbrace{\frac{C_{ES^2}}{2\mu u} E_{ab} S^a{}_c S^{cb}}_{\text{deformation due to spin}} - \underbrace{\frac{\mu_2}{4u^3} E_{ab} E^{ab} - \frac{2\sigma_2}{3u^3} B_{ab} B^{ab}}_{\text{tidal deformations}} + \ldots$$

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- $\{\mu, C_{ES^2}, \mu_2, \sigma_2\}$: constants, matched to single object
- Connection to Dixon's EOM: Bailey, Israel (1975)

$$J^{abcd} = 6 \frac{\partial R_M}{\partial R_{abcd}}$$

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Effective potential still valid . . .

. . . but now $\underline{m}(r) = \mu + \ldots$ (μ is constant by assumption)
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Dimensionless parameters

- Spin of the central black hole: $\hat{a}_1 := \frac{a}{M}, \quad |\hat{a}_1| \leq 1$
- Spin of the test-body: $\hat{a}_2 := \frac{S}{\mu^2}, \quad |\hat{a}_2| \leq 1$
- C_{ES^2} is already dimensionless
- Tidal deformation parameters: (test-body radius R)

$$k_2 := \frac{3\mu_2}{2R^5}, \qquad j_2 := \frac{48\sigma_2}{R^5}$$

- Dimensionless radii: $\hat{R} := \frac{R}{\mu}, \quad \hat{r} := \frac{r}{M}$
- Mass ratio: $q := \frac{\mu}{M}$

	$\hat{a}_2$	C_{ES^2}	k_2	j_2	$\hat{R}$
black hole	≤ 1	1	0	0	2
neutron star	$\lesssim 0.3$	~ 5	~ 0.1	~ -0.02	$\sim 5 \dots 7$

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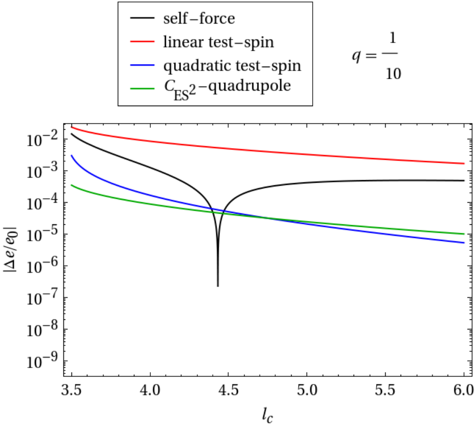
Results for Schwarzschild background

- Binding energy: $e(\hat{r}, J) = E/\mu - 1$
- Circular orbits $\rightsquigarrow \hat{r} \rightsquigarrow e(J)$
- Orbital angular momentum: $l_c = \frac{1}{M\mu}(J - S) \;\rightsquigarrow\; e(l_c)$



spin effects for $\hat{a}_2 = 1$, $C_{ES^2} = 1$

- Taylor-expansion: $e(l_c) = e_0(l_c) + \epsilon\, e_1(l_c) + \epsilon^2\, e_2(l_c) + \ldots$
- Scaling: $e_1 \propto q\hat{a}_2$, $e_2^{S^2} \propto -q^2\hat{a}_2^2$, $e_2^{C_{ES^2}} \propto -C_{ES^2} q^2 \hat{a}_2^2$

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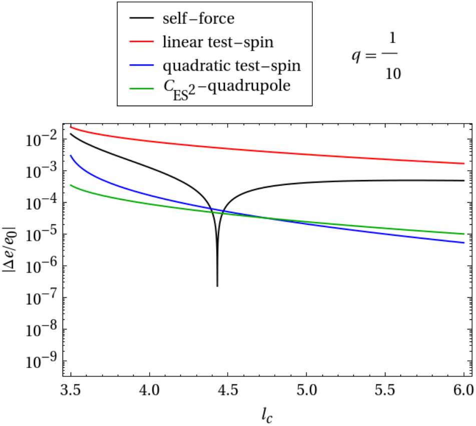

spin effects for $\hat{a}_2 = 1$, $C_{ES^2} = 1$

- Taylor-expansion: $e(l_c) = e_0(l_c) + \epsilon\, e_1(l_c) + \epsilon^2\, e_2(l_c) + \ldots$
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Results for Schwarzschild background

- Binding energy: $e(\hat{r}, J) = E/\mu - 1$
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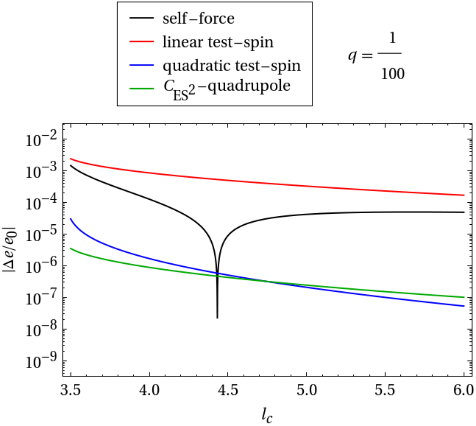

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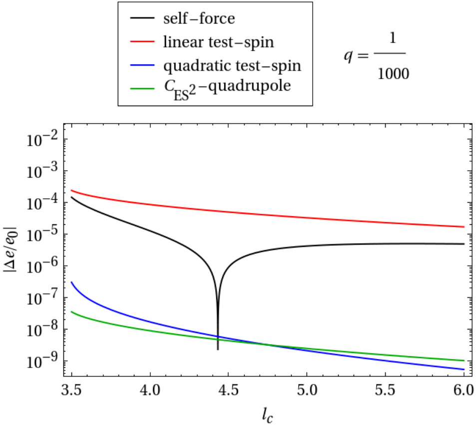

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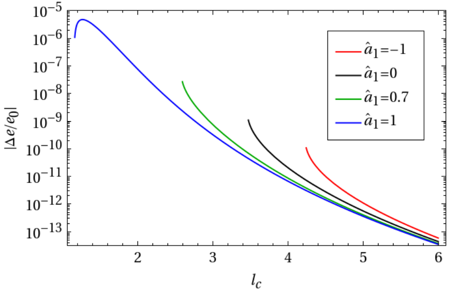
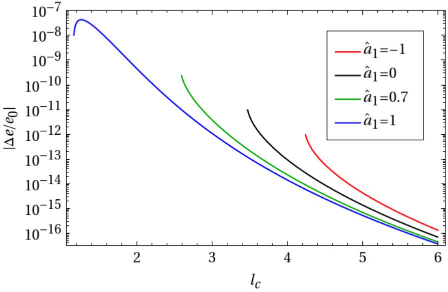
tidal effects for neutron stars and mass ratio $q = \frac{1}{50}$

$(k_2 = 0.1,\ j_2 = -0.01,\ \hat{R} = 5)$



gravito-electric tidal effects



gravito-magnetic tidal effects

- Scaling: $e_2^{k_2} \propto -k_2 q^4 \hat{R}^5$ $\qquad$ $e_2^{j_2} \propto j_2 q^4 \hat{R}^5$
- For $\hat{a}_1 = 1$ circular orbits are possible at the horizon!
- Limit due to tidal disruption: $\frac{e_2^{k_2}}{e_0} \lesssim \frac{k_2}{4\hat{R}} \sim 10^{-2}$

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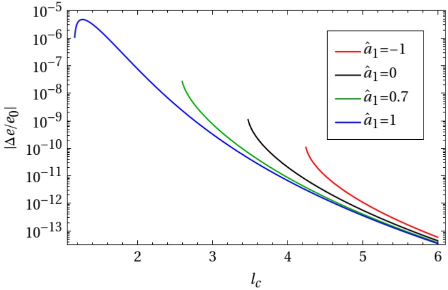

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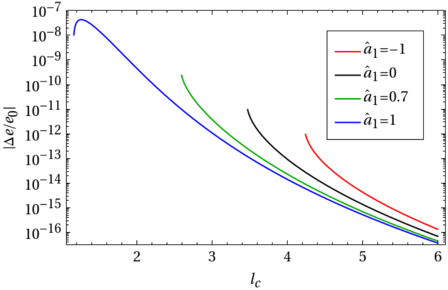

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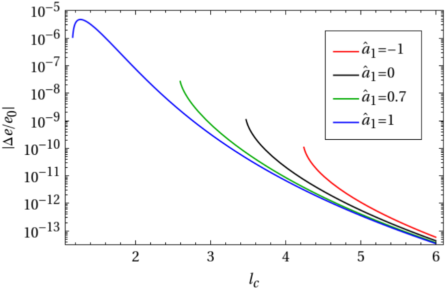

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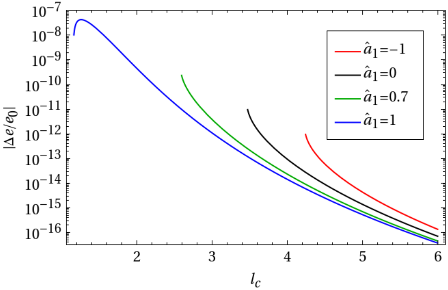

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Conclusions & Outlook

- Established connection between PN models and Dixon
- We identified relevance of some contributions due to the internal structure
- Spin-induced quadrupole effects scale like second-order self-force ($\sim q^2$)
- Gauge invariant $e(l_c)$: comparison with PN possible, also matching (EFT)
- Extension to comparable masses?
- Hamiltonian for generic orbits and spin orientations?

Thank you for your attention

and for support by CENTRA/IST and
the German Research Foundation DFG