Eddington inspired Born-Infeld gravity

prospects, problems, and extensions

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Outline

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Motivation

- Born-Infeld (BI) nonlinear electrodynamics:

$$S = \frac{1}{\kappa^2} \int d^4x \left[\sqrt{-\det(g_{ab} + \kappa F_{ab})} - \sqrt{-\det(g_{ab})} \right]$$

  [M. Born and L. Infeld, *Proc. R. Soc. A* **144** (1934) 425–451]

- Arises as low-energy effective theory from certain string theories.

  [E. Fradkin and A. A. Tseytlin, *Phys. Lett. B* **163** (1985) 123]

- Born-Infeld-Einstein gravity actions, e.g.:

$$S = \frac{2}{\gamma\kappa} \int d^4x \left[\sqrt{-\det(g_{ab} + \kappa R_{ab})} - \lambda\sqrt{-\det(g_{ab})} \right]$$

  [S. Deser and G. W. Gibbons, *Class. Quant. Grav.* **15** (1998) L35–L39]

- Eddington inspired Born-Infeld (EiBI) gravity: Palatini variation

  [M. Bañados and P. G. Ferreira, *Phys. Rev. Lett.* **105** (2010) 011101]

- Many similarities to Palatini $f(R)$!

  [T. P. Sotiriou and V. Faraoni, *Rev. Mod. Phys.* **82** (2010) 451–497]

- Metric formalism for BI-gravity actions inconsistent? (ghosts, instabilities)
- Palatini variation: more fundamental than metric formalism? (next slide)

Classification of gauge theories of gravity

from book: M. Blagojević, F.W. Hehl, "Gauge Theories of Gravitation", arXiv:1210.3775

[see Figure in arXiv:1210.3775]

Literature

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- I. Cho and H.-C. Kim, `arXiv:1302.3341 [gr-qc]`
- M. Bouhmadi-Lopez, C.-Y. Chen, and P. Chen, `arXiv:1302.5013 [gr-qc]`
- S. Rajagopal and A. Kumar, `arXiv:1303.6026 [gr-qc]`

Outline

Field Equations

D. N. Vollick, *Phys. Rev. D* **69** (2004) 064030

- Action of EiBI coupled to matter Ψ: $\quad \Lambda = \frac{\lambda - 1}{\kappa}, \quad g = \det(g_{ab})$

$$S[g, \Gamma, \Psi] = \frac{2}{\gamma\kappa} \int d^4x \left[\sqrt{-\det(g_{ab} + \kappa R_{ab}[\Gamma])} - \lambda\sqrt{-g} \right] + S_M[g, \Gamma, \Psi]$$

- Define auxiliary metric q_{ab} such that:

$$\Gamma^c_{ab} = \frac{1}{2} q^{cd} \left(\partial_a q_{bd} + \partial_b q_{ad} - \partial_d q_{ab} \right)$$

- Algebraic field equation:

$$\tau \left(g^{ab} - \frac{\gamma\kappa}{\lambda} T^{ab} \right) = q^{ab}$$

$$\tau := \sqrt{\frac{g}{q}}, \qquad q = \det(q_{ab}), \qquad T^{ab} := \frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta g_{ab}}$$

- Differential field equation:

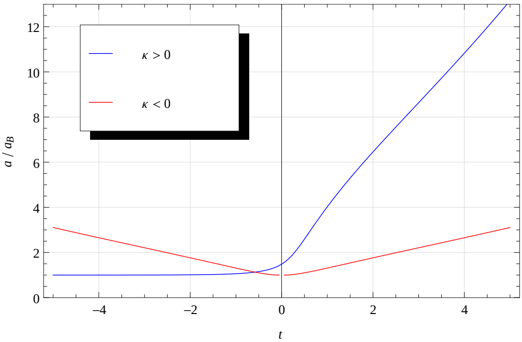
$$g_{ab} = \lambda q_{ab} - \kappa R_{ab}$$

- In the following, we set $\lambda = 1$ (i.e. $\Lambda = 0$) and $\gamma = 8\pi G$

Cosmology

M. Bañados and P. G. Ferreira, *Phys. Rev. Lett.* **105** (2010) 011101

- For $\kappa > 0$:
  - no big bang singularity!
  - loitering phase at early times
  - similar to Einstein universe
- For $\kappa < 0$:
  - no singularity!
  - bounce

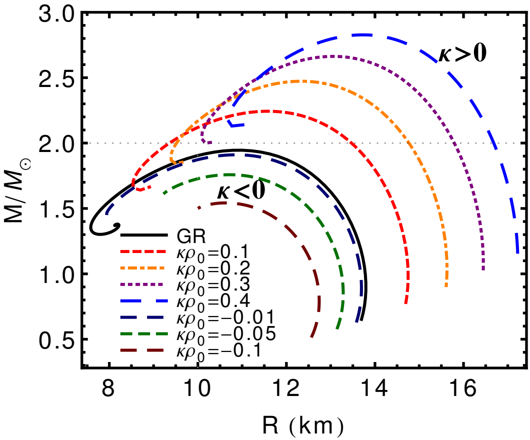


- Maximal density ρ_B, depends on sign(κ) and EOS
- Leads to minimal scale factor a_B
- How generic is the singularity avoidance in this theory?
- Similar singularity avoidance in Palatini $f(R)$

Compact Stars

P. Pani, V. Cardoso, and T. Delsate, *Phys. Rev. Lett.* **107** (2011) 031101

- For $\kappa > 0$:
  - repulsive effect
  - maximal mass increases
  - may save excluded EOS
- For $\kappa < 0$:
  - attractive effect
  - maximal mass decreases
- For $\kappa > 0$ the theory admits dust stars with EOS $P = 0$!



- Still a maximum mass exists: collapse to black hole not avoided
- At the surface the auxiliary metric q_{ab} is smooth
- But the "true" metric g_{ab} is maybe not even continuous!

Outline

Apparent source

Manipulate field equations:

$$\tau(g^{ab} - 8\pi G\kappa T^{ab}) = q^{ab} \qquad g_{ab} = q_{ab} - \kappa R_{ab}$$

Einstein equation for auxiliary metric

$$R^a{}_b = 8\pi G\left[\mathcal{T}^a{}_b - \frac{1}{2}\delta^a{}_b\mathcal{T}^c{}_c\right]$$

with the "apparent" stress tensor

$$\mathcal{T}^a{}_b = \tau T^a{}_b + \frac{\delta^a{}_b}{8\pi G}[\tau - 1 - 4\pi G\kappa\tau T]$$

$$\tau = \sqrt{\frac{g}{q}} = \frac{1}{\sqrt{\det(\delta^a{}_b - 8\pi G\kappa T^a{}_b)}}$$

Apparent source

Manipulate field equations:

$$\tau(\delta^a{}_b - 8\pi G\kappa T^a{}_b) = q^{ac}g_{cb} \qquad q^{ac}g_{cb} = \delta^a{}_b - \kappa R^a{}_b$$

Einstein equation for auxiliary metric

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Apparent source

Manipulate field equations:

$$\tau(\delta^a{}_b - 8\pi G\kappa T^a{}_b) = q^{ac} g_{cb} \;=\; q^{ac} g_{cb} = \delta^a{}_b - \kappa R^a{}_b$$

Einstein equation for auxiliary metric

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Apparent source

Manipulate field equations:

$$\tau(\delta^a{}_b - 8\pi G\kappa T^a{}_b) = q^{ac}g_{cb} \; = \; q^{ac}g_{cb} = \delta^a{}_b - \kappa R^a{}_b$$

Einstein equation for auxiliary metric

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Fluid: Modified EOS

$$\mathcal{T}^a{}_b = \tau T^a{}_b + \frac{\delta^a{}_b}{8\pi G}[\tau - 1 - 4\pi G\kappa\tau T]$$

$$\tau = \sqrt{\frac{g}{q}} = \frac{1}{\sqrt{\det(\delta^a{}_b - 8\pi G\kappa T^a{}_b)}}$$

- Interesting result for ideal fluid:

$$T^a{}_b = (\rho + P)\, u^a u_b + P\delta^a{}_b, \qquad u^a u^b g_{ab} = -1$$

$$\Rightarrow \mathcal{T}^a{}_b = (\rho_q + P_q)\, v^a v_b + P_q \delta^a_b, \qquad v_a v_b q^{ab} = -1$$

$$P_q = \frac{\tau}{2}(\rho - P) + \frac{\tau - 1}{8\pi G\kappa}, \quad \rho_q = \frac{\tau}{2}(\rho + 3P) - \frac{\tau - 1}{8\pi G\kappa}$$

$$\tau = \left[(1 + 8\pi G\kappa\rho)(1 - 8\pi G\kappa P)^3\right]^{-\frac{1}{2}}, \quad \rho_q + P_q = \tau(\rho + P)$$

- Similar: baryon number density, entropy, temperature
- For dust $P = 0$:

$$P_q = \pi G\kappa\rho_q^2 + \mathcal{O}(\rho_q^3)$$

Viability, Phenomenology, and Constraints

- Coupling between gravity and matter less explored
  $\rightarrow$ less constrained
- Theory is equivalent to general relativity in vacuum
- In vacuum EiBI can not be distinguished from GR with source $\mathcal{T}^a{}_b$
- Phenomenologically q_{ab}, $\mathcal{T}^a{}_b$, ρ_q, and P_q can be qualified as "apparent"
- Interesting: constraint on κ from observations of the sun (acoustic oscillation modes, neutrinos)

$$|\kappa| < 3 \cdot 10^5 \frac{\mathrm{m}^5}{\mathrm{s}^2\mathrm{kg}}$$

[J. Casanellas, P. Pani, I. Lopes, and V. Cardoso, *Astrophys. J.* **745** (2012) 15]

Energy Conditions (EC)

- EC used in the literature:

Null EC:	$\rho + P \geq 0$
Weak EC:	$\rho + P \geq 0,\ \rho \geq 0$
Strong EC, SEC:	$\rho + P \geq 0,\ \rho + 3P \geq 0$
Dominant EC:	$\rho \geq \lvert P \rvert$
Causal EC:	$\lvert \rho \rvert \geq \lvert P \rvert$

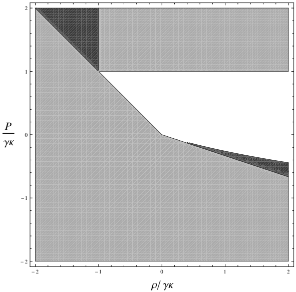
- Most important: Null EC
  Null EC violation associated with pathologies like traversable worm holes, warp drives, etc.
- Is Null EC fulfilled in apparent sector if it holds for the real EOS?

$$\rho_q + P_q = \tau(\rho + P), \qquad \tau \geq 0$$

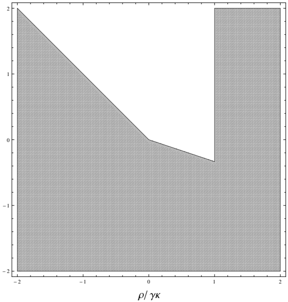
$\Rightarrow$ Yes, it is!

- Plots: illustrate Strong EC $\quad (\gamma = 8\pi G)$

Apparent SEC violation for real SEC, $\gamma\kappa > 0$



Apparent SEC violation for real SEC, $\gamma\kappa < 0$

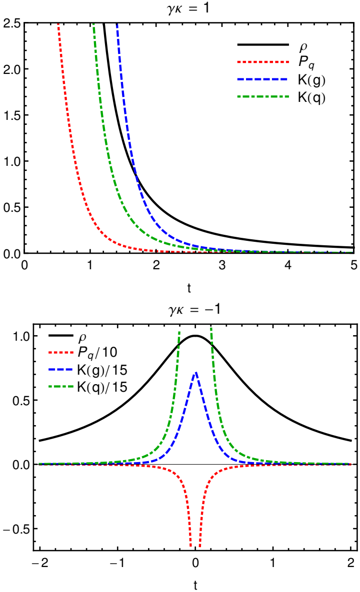


Analysis of Singularity Avoidance

- τ has poles for finite values of ρ and P:

$$\tau = \sqrt{\frac{g}{q}} = \frac{1}{\sqrt{(1 + 8\pi G\kappa\rho)(1 - 8\pi G\kappa P)^3}}$$

- Pole for $8\pi G\kappa P \to 1$:
  - Can happen for $\kappa > 0$
  - Maximal pressure $P_{\text{max}} = 1/8\pi G\kappa$ (corresponding to singular $\rho_q \propto \tau\rho\ldots$)
  - EOS is considerably softened
- Pole for $8\pi G\kappa\rho \to -1$:
  - Can happen for $\kappa < 0$
  - Max. energy density $\rho_{\text{max}} = 1/8\pi G|\kappa|$
  - EOS is considerably hardened
- Counterexample: dust universe (plots)
- ”Usual” EOS should avoid singularities!



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Problems and Extensions

P. Pani and T. P. Sotiriou, *Phys. Rev. Lett.* **109** (2012) 251102

- Theory is well behaved in q_{ab} sector
- But Ricci scalar $R[g]$ at surface of stars is singular if (near the surface)

$$P = K\rho_0^\Gamma \qquad \text{with} \qquad \Gamma > 3/2$$

- Generic problem, differential structure of theory in terms of g_{ab}: higher order derivatives of matter in source
- The metric g_{ab} is too sensitive to sharp matter profiles
- Same problem appears for Palatini $f(R)$
- But implications are discussed controversially in Palatini $f(R)$ gravity
- Can probably be cured by adding further degrees of freedom: torsion, nonmetricity, ...

Extensions from bimetric action approach?

- A bimetric linearization of the action reads:

$$S_G = \frac{1}{8\pi G} \int d^4x \sqrt{-q} \left[R[q] - 2\frac{\lambda}{\kappa} + \frac{1}{\kappa} \left(q^{ab} g_{ab} - 2\sqrt{\frac{g}{q}} \right) \right] + S_M[g]$$

- Stringy anlogon: from Nambu-Goto to Polyakov action
- No ghosts!
- Only the metric coupling to matter is measurable
- "Cutoff" $1/8\pi G\kappa$ appears as coupling parameter
- Starting point to modify the gravity-matter coupling?
- Similar bimetric action used in asymptotic safety scenario
  E. Manrique, M. Reuter, and F. Saueressig, *Annals Phys.* **326** (2011) 440–462
- More complicated bimetric actions appear in New Massive Gravity
  S. Hassan and R. A. Rosen, *JHEP* **1107** (2011) 009
  S. Hassan and R. A. Rosen, *JHEP* **1202** (2012) 126

Outline

Conclusions

- Exiting features of EiBI:
  - BI structure may originate from quantized gravity
  - Palatini variation is very natural
  - Some singularities are avoided
  - Dust stars (pressureless) for $\kappa > 0$
  - Coupling between gravity and matter less explored/constrained
  - Interesting phenomenology, as it deviates from GR only inside matter
- Problems:
  - Maximum NS mass vs. singularity avoidance
  - Similar to Palatini $f(R)$, same problems, e.g.:
  - Problems at surface/phase transitions, like singular curvature
- Possible extensions:
  - Modification using bimetric action
  - Modification by relaxing conditions on connection: torsion, nonmetricity

Thank you for your attention

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