Effective action for compact objects

and universal relations

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Seminar at Rome University La Sapienza

Outline

Experiments

Pulsars and radio astronomy:

pics/doublepulsar

Double Pulsar (MPI for Radio Astronomy)

$\rightsquigarrow$

pics/ska

Square Kilometre Array (SKA)

Gravitational wave detectors:

pics/ligo

Advanced LIGO

pics/lisa

eLISA space mission

Pulsar Timing Array

γ-rays, X-rays, ...

pics/xraybin

e.g. large BH spins in X-ray binaries

Neutron stars and black holes

pics/neutronstar

Neutron star picture by D. Page

www.astroscu.unam.mx/neutrones/

„Lab“ for various areas in physics

- magnetic field, plasma
- crust (solid state)
- superfluidity
- superconductivity
- unknown matter in core
  condensate of quarks, hyperons, kaons, pions, . . . ?

  accumulation of dark matter ?

Black holes are simpler, but:

- strong gravity
- horizon

analytic models?

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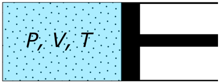
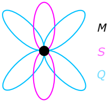
Models for multipoles of compact objects

Starting point: single object, e.g., neutron star



$\longleftrightarrow$



state variables (p, V, T)	$\longleftrightarrow$	multipoles (m, S, Q)
thermodynamic potential	$\longleftrightarrow$	dynamical mass $\mathcal{M}$
correlation	$\longleftrightarrow$	response

Idea

Multipoles describe compact object on macroscopic scale

- Higher multipole order $\rightarrow$ smaller scales $\rightarrow$ more (internal) structure
- Multipoles describe the gravitational field and interaction
- Multipoles of neutron stars fulfill universal (EOS independent) relations

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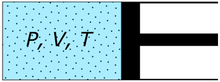

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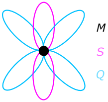

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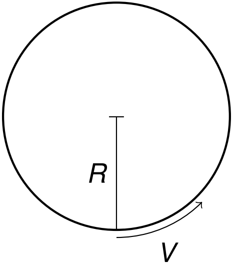
Two Facts on Spin in Relativity

1. Minimal Extension



- ring of radius R and mass M
- spin: $S = R\,M\,V$
- maximal velocity: $V \leq c$
  $\Rightarrow$ minimal extension:

$$R = \frac{S}{MV} \geq \frac{S}{Mc}$$

2. Center-of-mass

- now moving with velocity v
- relativistic mass changes inhom.
- frame-dependent center-of-mass
- need spin supplementary condition:

$$\text{e.g.,} \quad S^{\mu\nu} p_\nu = 0$$

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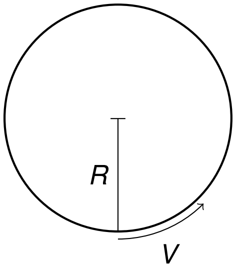

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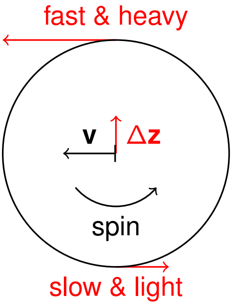

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Spin gauge symmetry in an action principle

choice of center should be physically irrelevant
$\Rightarrow$ gauge symmetry?

Construction of an action principle in flat spacetime:

- introduce orthonormal corotating frame $\Lambda_1{}^\mu$, $\Lambda_2{}^\mu$, $\Lambda_3{}^\mu$
- complete it by a time direction $\Lambda_0{}^\mu$ such that

$$\eta^{AB} \Lambda_A{}^\mu \Lambda_B{}^\nu = \eta^{\mu\nu}$$

- realize that $\Lambda_0{}^\mu$ is redundant/gauge since
  one can boost $\Lambda_A{}^\mu$ such that $\text{Boost}(\Lambda_0) \propto p$ (p_μ: linear momentum)
- find symmetry of the kinematic terms in the action:

$$p_\mu \dot{z}^\mu + \frac{1}{2} S_{\mu\nu} \Lambda_A{}^\mu \dot{\Lambda}^{A\nu}$$

$$z^\mu \to z^\mu + \Delta z^\mu$$

$$S_{\mu\nu} \to S_{\mu\nu} + p_\mu \Delta z_\nu - \Delta z_\mu p_\nu$$

$$\Lambda \to \text{Boost}_{p \to \Lambda_0 + \epsilon} \text{Boost}_{\Lambda_0 \to p} \Lambda$$

- find invariant quantities, minimal coupling

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Point Particle Action in General Relativity

Westpfahl (1969); Bailey, Israel (1975); Porto (2006); Levi & Steinhoff (2014)

Minimal coupling to gravity, in terms of invariant position:

$$S_{\text{PP}} = \int d\sigma \left[p_\mu \frac{Dz^\mu}{d\sigma} - \frac{p_\mu S^{\mu\nu}}{p_\rho p^\rho} \frac{Dp_\nu}{d\sigma} + \frac{1}{2} S_{\mu\nu} \Lambda_A{}^\mu \frac{D\Lambda^{A\nu}}{d\sigma} - \frac{\lambda}{2} \mathcal{H} - \chi^\mu \mathcal{C}_\mu \right]$$

- constraints: $\mathcal{H} := p_\mu p^\mu + \mathcal{M}^2 = 0$, $\mathcal{C}_\mu := S_{\mu\nu}(p^\nu + p\Lambda_0{}^\mu)$
- Dynamical mass $\mathcal{M}$ includes multipole interactions

Application: post-Newtonian approximation

- for bound orbits
- one expansion parameter, $\epsilon_{\text{PN}} \sim \frac{v^2}{c^2} \sim \frac{GM}{c^2 r} \ll 1$ (weak field & slow motion)

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Spin and Gravitomagnetism

Interaction with gravito-magnetic field $A_i \approx -g_{i0}$:

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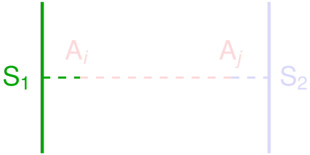
$\rightarrow$ universal for all objects!



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- Here: $\delta_a = \delta(\vec{x} - \vec{z}_a)$, $r_a = |\vec{x} - \vec{z}_a|$
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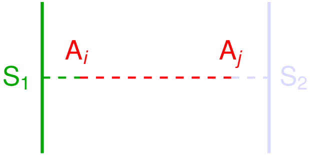

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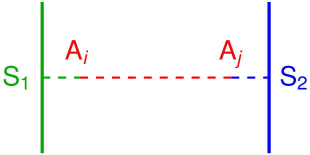

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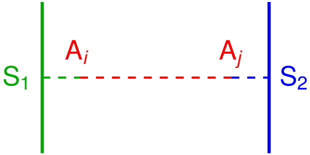

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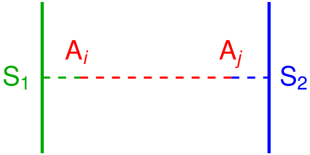

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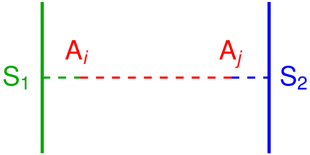

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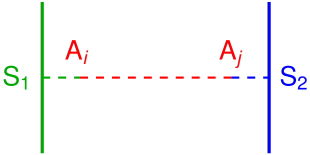

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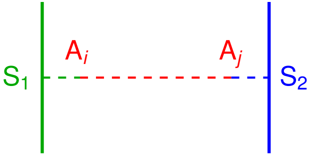

$$\int d^3x \, \frac{1}{2} S_1^{ki} \partial_k \delta_1 \; 16\pi G \delta_{ij} \Delta^{-1} \; \frac{1}{2} S_2^{lj} \partial_l \delta_2$$

$$= \int d^3x \, \frac{1}{2} S_1^{ki} \partial_k \delta_1 \; (-2) G S_2^{li} \partial_l \left(\frac{1}{r_2} \right)$$

$$= G S_1^{ki} S_2^{li} \partial_k \partial_l \left(\frac{1}{r_2} \right) \Big|_{\vec{x}=\vec{z}_1}$$

- Leading-order $S_1 S_2$ potential
- Here: $\delta_a = \delta(\vec{x} - \vec{z}_a)$, $r_a = |\vec{x} - \vec{z}_a|$
- Diagrams encode integrals: Feynman rules [e.g. arXiv:1501.04956]
- Analogous to spin interaction in atomic physics
- Status: NNLO

Outline

Quadrupole Deformation due to Spin

for neutron stars. See e.g. Laarakkers, Poisson (1997); Porto, Rothstein (2008)

Coupling in the effective point-particle action:

$$\mathcal{M}^2 = m^2 + C_{ES^2} E_{\mu\nu} S^{\mu\alpha} S^{\nu}{}_{\alpha} + \ldots \qquad E_{\mu\nu} := -R_{\mu\alpha\nu\beta} \frac{p^\alpha p^\beta}{p_\rho p^\rho}$$

- C_{ES^2} = dim.-less quadrupole $\bar{Q}$:

$$C_{ES^2} = \bar{Q} := \frac{Q}{ma^2} \approx \text{const}$$

where $a = \frac{S}{m^2}$

- $\bar{Q} = 4 \ldots 8$ for $m = 1.4 M_{\text{Sun}}$
  EOS dependent!
- For black holes $\bar{Q} = 1$
- effective theory to hexadecapole order: Levi, JS (2014) & (2015)
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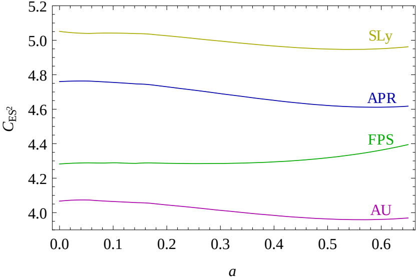

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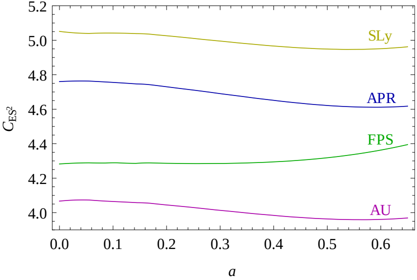

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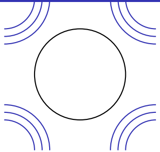
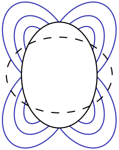
Dynamic tides: External field and response



linear response

$\xrightarrow{F}$



external quadrupolar field $\longrightarrow$ deformation $\longrightarrow$ quadrupolar response

Newtonian:

$r^{\ell+1}$ $\qquad\qquad$ $r^{-\ell}$

relativistic, adiabatic $\omega = 0$: $\qquad$ [Hinderer & Flanagan (2008)]

$r^{\ell+1}\ {}_2F_1(\dots; 2m/r)$ $\qquad\qquad$ $r^{-\ell}\ {}_2F_1(\dots; 2m/r)$

relativistic, generic ω:

X^{ℓ}_{MST} $\qquad\qquad$ $X^{-\ell-1}_{\text{MST}}$

where [Mano, Suzuki, Takasugi, PTP **96** (1996) 549]

$$X^{\ell}_{\text{MST}} = e^{-i\omega r}(\omega r)^{\nu}\left(1 - \frac{2m}{r}\right)^{-i2m\omega}\sum_{n=-\infty}^{\infty}\dots\times\left[\frac{r}{2m}\right]^{n}{}_2F_1(\dots; 2m/r)$$

Renormalized angular momentum, transcendental number: $\nu = \nu(\ell, m\omega)$

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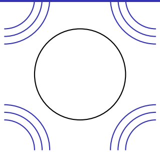

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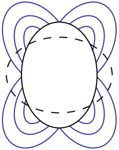

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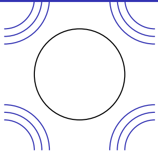

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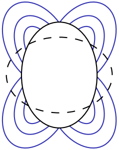

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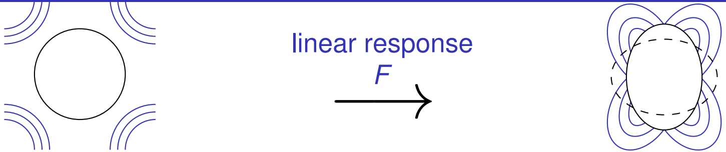

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Identification of external field and response by considering generic ℓ (analytic continuation)

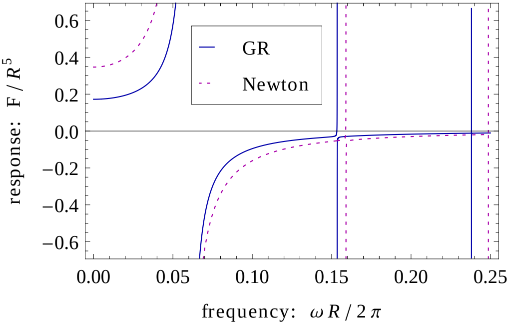
Dynamic tides: Results

Chakrabarti, Delsate, JS (2013)

Fit for the response $Q = F\,E$:

$$F(\omega) \approx \sum_n \frac{I_n^2}{\omega_n^2 - \omega^2}$$

(exact in Newtonian case)



$\Rightarrow$ dynamical mass augmented by harmonic oscillators q_n,p_n:

$$\mathcal{M} = m + \sum_n (p_n^2 + \omega_n^2 q_n^2 + 2 I_n q_n E) + \ldots ,$$

- poles $\Rightarrow$ resonances at mode frequencies ω_n
- modes appear as normal modes instead of QNM
- Relativistic overlap integrals: I_n
- $F(\omega = 0)$ is Love number λ

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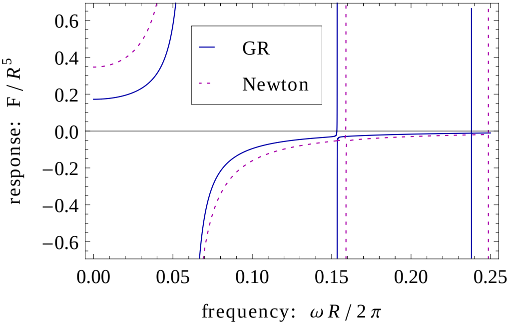

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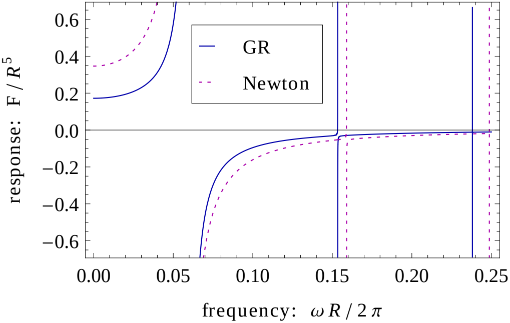

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pics/resonance

resonance (Tacoma Bridge)

Outline

Universal relation: I Love Q!

K. Yagi, N. Yunes, Science **341**, 365 (2013) [plots taken from there]

universal $\equiv$ independent of equation of state

(approximately) universal relation between dimensionless

moment of inertia $\bar{I}$, Love number $\bar{\lambda}$, spin-induced quadrupole $\bar{Q}$

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universality: existence is natural due to scaling

earlier work:

- $I(c)$: J. Lattimer and M. Prakash, ApJ. **550** (2001) 426
- f-mode: L. Tsui and P. Leung, MNRAS **357** (2005) 1029
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But surprising: 1% accuracy of I-Love-Q

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Universal relations for fast rotation

Do relations hold in more realistic situation? $\rightarrow$ beyond slow rotation?

- No. Doneva, Yazadjiev, Stergioulas, Kokkotas, ApJ Lett. **781** (2014) L6
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- Yes! Chakrabarti, Delsate, Gürlebeck, Steinhoff, PRL **112** (2014) 201102

$\bar{I}$-$\bar{Q}$ relation depends on a parameter!
Different choices work:

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Again, holds within 1%

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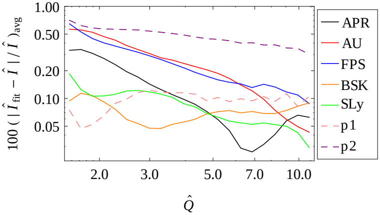

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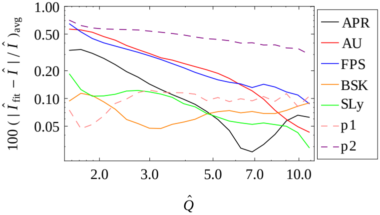

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- should re-fit for optimal accuracy estimate

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$\hat{f} = 200mf$

dashed lines: slow rotation approximation

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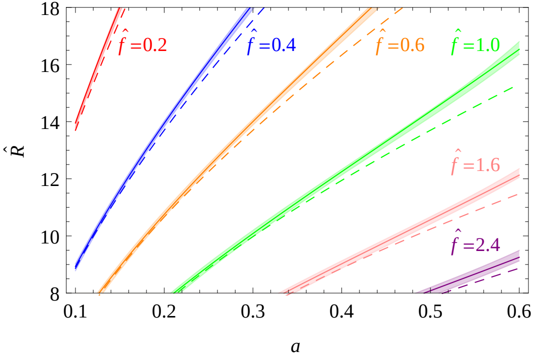

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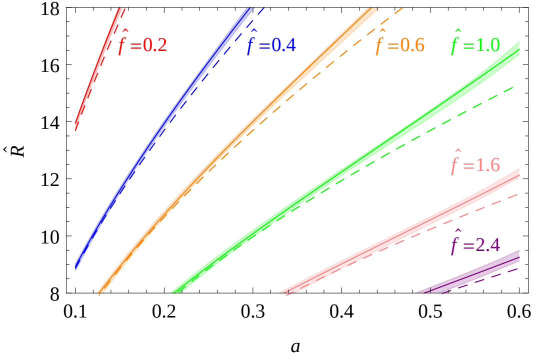

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Conclusions

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- and (part of) the mode spectrum
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Large scale "thermodynamic" picture very useful for binaries & GW

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