Analytic approximations for the gravitational interaction of compact objects

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Outline

1. Motivation
2. Multipole approximation
3. Applications
4. Tidal effects beyond the adiabatic case
5. Future Plans

Motion in General Relativity

Motivation:

- Gravitational wave experiments: Advanced LIGO in 2015
  (possibly $>$40 detections of binary NS mergers per year)
- Pulsar timing via radio astronomy: double pulsar, SKA, ...
  (also optical: WD+WD binary J0651+2844)
- Formation of supermassive BH vs. gravitational recoil ("kick")
- Gravity Probe B
- SgrA*, LRR, Planetary motion, ...

⇒ most gravity experiments require to study the motion!

Possibilities:

- extreme mass ratio approximation, self-force
- Full numeric simulations (still computationally very expensive)
- post-Minkowskian approximation (weak field)
- post-Newtonian (PN) approximation (weak field & slow motion)

⇒ when the parameter space is large, analytic methods are invaluable.

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Multipole Approximation

Equations of motion:

$$\frac{Dp_\mu}{d\tau} = 0 - \frac{1}{2} R_{\mu\rho\beta\alpha} u^\rho S^{\beta\alpha} - \frac{1}{6} R_{\nu\rho\beta\alpha;\mu} J^{\nu\rho\beta\alpha} + \dots$$

$$\frac{DS^{\mu\nu}}{d\tau} = 2p^{[\mu} u^{\nu]} + \frac{4}{3} R_{\alpha\beta\rho}{}^{[\mu} J^{\nu]\rho\beta\alpha} + \dots$$

Singular energy momentum tensor, $\delta_{(4)} = \delta(x^\sigma - z^\sigma)$:

$$\sqrt{-g} T^{\mu\nu}(x^\sigma) = \int d\tau \Big[u^{(\mu} p^{\nu)} \delta_{(4)} + \left(u^{(\mu} S^{\nu)\alpha} \delta_{(4)} \right)_{||\alpha} + \frac{1}{3} R_{\alpha\beta\rho}{}^{(\mu} J^{\nu)\rho\beta\alpha} \delta_{(4)} - \frac{2}{3} \left(J^{\mu\alpha\beta\nu} \delta_{(4)} \right)_{||(\alpha\beta)} + \dots \Big]$$

- Geodesic equation: momentum p_μ
- Mathisson (1937), Papapetrou (1951): spin / dipole $S^{\mu\nu}$
- Dixon (~1974): quadrupole $J^{\mu\nu\alpha\beta}$, ...
- EOM for p_μ and $S^{\mu\nu}$ follow from theory! $T^{\mu\nu}{}_{;\nu} = 0 \rightsquigarrow$ EOM

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Spin Action in GR

Westpfahl (1969); Bailey, Israel (1975); Porto (2006); Steinhoff, Schäfer (2009)

- Angular 4-velocity tensor from Lorentz matrix $\Lambda^{A\nu}$:

$$\Omega^{\mu\nu} = -\Omega^{\nu\mu} = \Lambda_A{}^{\mu} \frac{D\Lambda^{A\nu}}{d\tau}$$

- Lagrangian with minimal coupling:

$$L = m\sqrt{-u_\mu u^\mu} + \underbrace{\frac{1}{2} S_{\mu\nu} \Omega^{\mu\nu}}_{\rightsquigarrow \frac{1}{2} S^{ij} \partial_i A_j} + \ldots$$

- $m = \text{const}$
- Valid to linear order in spin
- Angular velocity vector is $\Omega^i = \frac{1}{2} \epsilon_{ijk} \Omega^{jk}$. Analogous for spin.
- Gravito-magnetic field $A_i \approx -g_{i0}$

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Canonical Variables to Linear Order in Spin

Steinhoff, Schäfer (2009)

- Method: transform action into the form $\int dt(\dot{q}\,p - H)$
- Flat spacetime: Newton-Wigner center and spin are canonical
- Canonical $\hat{z}^i$, $\hat{S}_{ij}$, and $\hat{\Lambda}^{ij}$ are "simple" generalizations of flat space case
- Canonical matter momentum $\hat{p}_i$:

$$p_i = \hat{p}_i + \frac{1}{2}\hat{S}_{kj}\Gamma^{kj}{}_i + \ldots$$

cf. electrodynamics: $p_i = \hat{p}_i - qA_i$

- Test-spin Hamiltonian [Barusse, Racine, Buonanno, arXiv:0907.4745]: insert background metric into action, transform to canonical variables
- Even the canonical field momentum changes (self-gravitating case)

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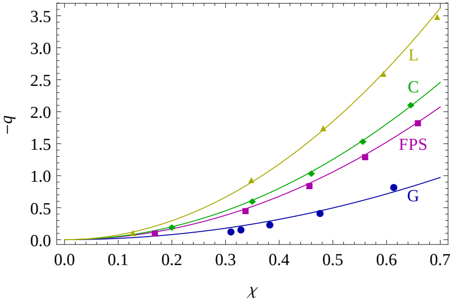
Quadrupole Deformation due to Spin

for neutron stars: Laarakkers, Poisson (1997)

- Here $m = 1.4 M_{\odot}$
- Dim.-less mass quadrupole: q
- Dim.-less spin: χ
- Quadratic fit is extremely good:

$$-q \approx C_{ES^2} \chi^2$$

- $C_{ES^2} = 4.3 \ldots 7.4$, EOS dependent
- Also depends on mass
- For black holes $C_{ES^2} = 1$



see Laarakkers, Poisson gr-qc/9709033

- modeled by nonminimal couplings in the action [Porto, Rothstein (2008)]
- higher multipoles: Pappas, Apostolatos (2012)
- similar for tidal deformation

Post-Newtonian results so far

from various authors with different methods

for maximally rotating objects: $S = \frac{Gm^2\chi}{c}$ $\quad \chi = 1$

order	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
H^{N}								
PM	$+ H^{1\mathrm{PN}}$		$+ H^{2\mathrm{PN}}$	$+ H^{2.5\mathrm{PN}}$	$+ H^{3\mathrm{PN}}$	$+ H^{3.5\mathrm{PN}}$	$+ H^{4\mathrm{PN}}$	$+ H^{4.5\mathrm{PN}}$
SO		$+ H^{\mathrm{LO}}_{\mathrm{SO}}$		$+ H^{\mathrm{NLO}}_{\mathrm{SO}}$		$+ H^{\mathrm{N^2LO}}_{\mathrm{SO}}$	$+ H^{\mathrm{LO,R}}_{\mathrm{SO}}$	$+ H^{\mathrm{N^3LO}}_{\mathrm{SO}}$
S_1^2			$+ H^{\mathrm{LO}}_{S_1^2}$		$+ H^{\mathrm{NLO}}_{S_1^2}$		$+ H^{\mathrm{N^2LO}}_{S_1^2}$	$+ H^{\mathrm{LO,R}}_{S_1^2}$
S_1S_2			$+ H^{\mathrm{LO}}_{S_1S_2}$		$+ H^{\mathrm{NLO}}_{S_1S_2}$		$+ H^{\mathrm{N^2LO}}_{S_1S_2}$	$+ H^{\mathrm{LO,R}}_{S_1S_2}$
spin^3						$+ H^{\mathrm{LO}}_{S^3}$		$+ H^{\mathrm{NLO}}_{S^3}$
spin^4							$+ H^{\mathrm{LO}}_{S^4}$	
$\vdots$								$\ddots$

H known   EOM known   for Black Holes   not known (yet)

Radiation field known to 2PN order, multipoles to 2.5PN order.

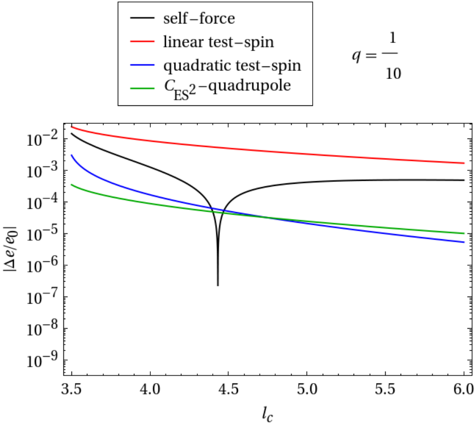
Motion in Schwarzschild background

Steinhoff, Puetzfeld (2012); similar model: Bini, Geralico (2013)

- Conserved quantities:
  $E_{\partial_t}, E_{\partial_\phi}, S, m$
- Circular orbits, aligned spin
- SSC: $S^{\mu\nu} p_\nu = 0$

$\Rightarrow p_\nu$, $S^{\mu\nu}$ fixed algebratically!

- Binding energy:
  $e(l_c) = E_{\partial_t}/m - 1$
- Orbital angular momentum: l_c



spin effects for $\hat{a}_2 = 1$, $C_{ES^2} = 1$

- Multipole expansion: $e(l_c) = e_0(l_c) + e_1(l_c) + e_2(l_c) + \dots$
- Scaling: $e_1 \propto q\hat{a}_2$, $e_2^{S^2} \propto -q^2\hat{a}_2^2$, $e_2^{C_{ES^2}} \propto -C_{ES^2} q^2 \hat{a}_2^2$
- Future: derive Hamiltonian for generic orbits

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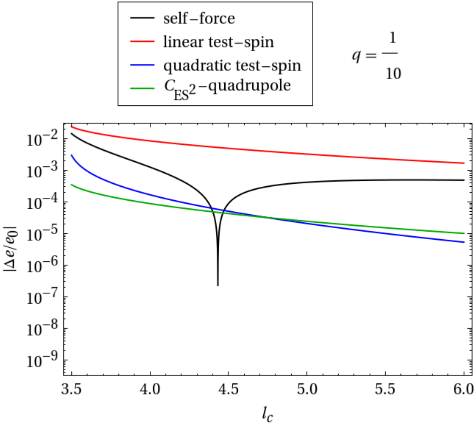

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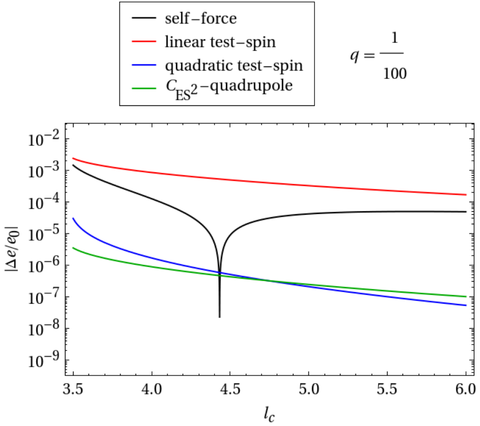

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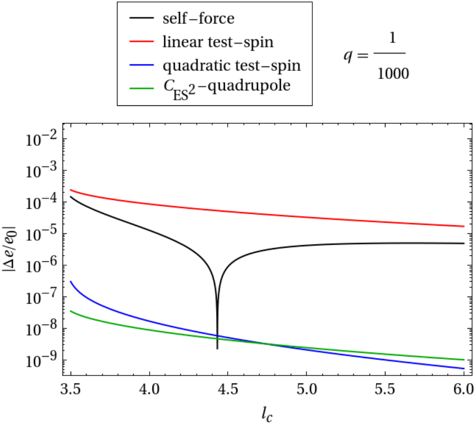

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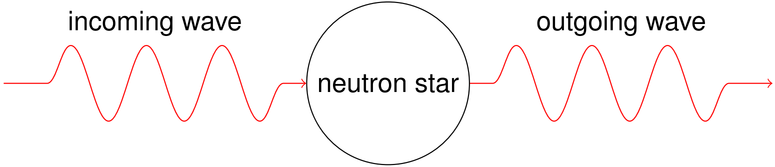
Tidal effects beyond the adiabatic case

with S. Chakrabarti and T. Delsate, arXiv:1304.2228 [gr-qc]

- Adiabatic tidal effects may not be sufficient [Maselli et.al. (2012)]
- Resonances of orbital motion and oscillation modes of the object
- Idea: response function for Q^{ab} [Goldberger, Rothstein, hep-th/0511133]

$$Q^{ab}(t) = -\frac{1}{2} \int dt' \ F^{ab}{}_{cd}(t, t') \, E^{cd}(t')$$

- Analysis in Fourier space:



- Analogy to optics: refractive index is response, need phase shift
  also: absorption from imaginary part of $F(\omega)$

Methods and results

with S. Chakrabarti and T. Delsate, arXiv:1304.2228 [gr-qc]

- Method: inhomogeneous Regge-Wheeler equation

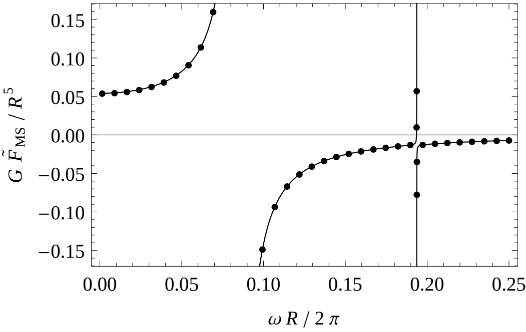
$$\frac{d^2X}{dr_*^2} + \left[\left(1 - \frac{2M}{r}\right)\frac{l(l+1) - \frac{6M}{r}}{r^2} + \omega^2\right] X = S,$$

- Analytic solutions for hom. equation are known: series of ${}_1F_1$ and ${}_2F_1$ [Mano, Suzuki, Takasugi, arXiv:gr-qc/9605057]

- Fit for the response:

$$F(\omega) = \sum_n \frac{q_n^2}{\omega_n^2 - \omega^2}$$

- Just like in Newtonian case!
- ω_n are the mode frequencies
- q_n related to overlap integrals
- Matching scale is fitted, too



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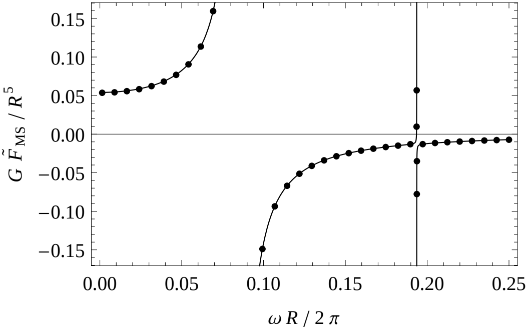

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Future Plans

Spin corrections to radiation field:

- Restricted waveform (phase) to 4PN order (now: 2.5PN and 3.5PN SO)
- Full waveform (phase and amplitude) to 3.5PN (now: 2PN)

Dynamic multipoles and tides:

- More realistic NS models: rotation, crust, ...
- Resonances with orbital motion
- Instabilities of modes, shattering of crust, connection to GRB, ...

More Hamilton functions:

- Test-particle Hamiltonian for small q including quadrupole:
  - test-NS in the field of a Kerr BH or a "massive" NS
  - Extension to comparable masses?
- post-Newtonian Hamiltonians to 4.5PN:
  - $H^{\mathrm{LO}}_{S^3}$ and $H^{\mathrm{LO}}_{S^4}$ for (neutron) stars
  - $H^{\mathrm{N^2LO}}_{S_1^2}$ at 4PN
  - $H^{\mathrm{N^3LO}}_{\mathrm{SO}}$ and $H^{\mathrm{NLO}}_{S^3}$ at 4.5PN (later)

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- post-Newtonian Hamiltonians to 4.5PN:
  - $H^{\mathrm{LO}}_{S^3}$ and $H^{\mathrm{LO}}_{S^4}$ for (neutron) stars
  - $H^{\mathrm{N^2LO}}_{\mathrm{S}_1^2}$ at 4PN
  - $H^{\mathrm{N^3LO}}_{\mathrm{SO}}$ and $H^{\mathrm{NLO}}_{S^3}$ at 4.5PN (later)

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